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Exercises for Algebraic Geometry 1

Winter term 2017/2018

Exercise sheet 6

Exercise 1. Let R be a commutative ring. Show that

$$R[x_1, \ldots, x_m] \otimes_R R[y_1, \ldots, y_n] \cong R[x_1, \ldots, x_m, y_1, \ldots, y_n]$$

as R-algebras.

Exercise 2. Let M,N and P be R-modules over a commutative ring R. Show that:

(1) $M \otimes_R R \cong M$

(2) $M \otimes_R N \cong N \otimes_R M$

(3) $(M \otimes_R N) \otimes_R P \cong M \otimes_R (N \otimes_R P)$

Observe that these isomorphisms are isomorphisms as R-algebras if M, N and P are R-algebras.

Exercise 3. Let $f : X \to Y$ be a morphism between affine varieties, and let $f^* : A(Y) \to A(X)$ be the corresponding map of k-algebras. Which directions of the following statements are true?

(1) If $P \in X$ and $Q \in Y$, then $f(P) = Q$ if and only if $(f^*)^{-1}(I(P)) = I(Q)$.

(2) f^* is injective if and only if f is surjective.

(3) f^* is surjective if and only if f is injective.

(4) f is an isomorphism if and only if f^* is an isomorphism.

Exercise 4. Which of the following algebraic sets are isomorphic over the complex numbers?

(1) $\mathbb{A}^1$

(2) $Z(x^2 + y^2) \subseteq \mathbb{A}^2$

(3) $Z(x^2 - y^3) \subseteq \mathbb{A}^2$

(4) $Z(xy) \subseteq \mathbb{A}^2$

(5) $Z(y^2 - x^3 - x^2) \subseteq \mathbb{A}^2$

(6) $Z(y - x^2, z - x^3) \subseteq \mathbb{A}^3$

We will discuss the exercises on **Thursday**, **23.11.2017**!