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Exercises for Algebraic Geometry 1

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Exercise sheet 7

Exercise 1. Show that the prevariety $\mathbb{P}^1$ is a variety.

Exercise 2. Let X and Y be prevarieties with affine covers $\{U_i\}$ and $\{V_j\}$, respectively.

(1) Using Lemma 2.4.7 in Gathmann's Skript, construct the product prevariety $X \times Y$ by glueing the affine varieties $U_i \times V_j$ together.

(2) Show that there are projective morphisms $\pi_X : X \times Y \to X$ and $\pi_Y : X \times Y \to Y$ satisfying the *universal property for products*:

Given morphisms $f : Z \to X$ and $g : Z \to Y$ from any prevariety Z, there is a unique morphism $h : Z \to X \times Y$ such that $f = \pi_X \circ h$ and $g = \pi_Y \circ h$.

Exercise 3. Show that:

(1) Every isomorphism $f : \mathbb{A}^1 \to \mathbb{A}^1$ is of the form $f(x) = ax + b$ for some $a, b \in k$, $a \neq 0$.

(2) Every isomorphism $f : \mathbb{P}^1 \to \mathbb{P}^1$ is of the form $f((x : y)) = (ax + by : cx + dy)$ for some $a, b, c, d \in k$.

(3) Given three distinct points $P_1, P_2, P_3 \in \mathbb{P}^1$ and three distinct points $Q_1, Q_2, Q_3 \in \mathbb{P}^1$, there is a unique isomorphism $\mathbb{P}^1 \to \mathbb{P}^1$ such that $f(P_i) = Q_i$ for $i = 1, 2, 3$.

Exercise 4. If U is an open subset of a prevariety X and $f : U \to \mathbb{P}^1$ a morphism, is it always true that f can be extended to a morphism $\tilde{f} : X \to \mathbb{P}^1$?
