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Exercises for Algebraic Geometry 1

Winter term 2017/2018

Sheet 11

Exercise 1. Consider the Segre embedding

$$\sigma : \mathbb{P}^1 \times \mathbb{P}^1 \longrightarrow \mathbb{P}^3,$$
$$((x_0 : x_1), (y_0 : y_1)) \longmapsto (x_0y_0 : x_0y_1 : x_1y_0 : x_1y_1)$$

and its image $S \subseteq \mathbb{P}^3$. Let $y \in \mathbb{P}^1$. Show that the image of the line $\mathbb{P}^1 \times \{y\}$ in S cannot be obtained as the zero locus of a single form on S.

Exercise 2. Let $Y \subseteq \mathbb{A}^n$ be an affine variety, $m := \dim Y$, and let $y \in Y$. Show that there are affine linear forms $g_1, \ldots, g_m$ on $\mathbb{A}^n$ and an open neighborhood $V \subseteq Y$ of y such that

$$Z(g_1, \ldots, g_m) \cap V = \{y\}.$$

Exercise 3. Let k be an algebraically closed field. Consider the morphism

$$\varphi : k^{2\times 2} \longrightarrow k^{2\times 2},$$
$$A \longmapsto A^2.$$

(a) Is φ surjective? Is it dominant?

(b) What is the cardinality of a typical fiber? What exactly does *typical* mean?

(c) Describe all possible types of fibers.
(Hint: use the Jordan normal form.)

We discuss these exercises on **18.01.2018**!