

Algebra II: Commutative Algebra

4h lecture, TU Berlin, SS 2020, Prof. P. Bürgisser

Chap. 1. Rings and Ideals

- 1.1 Rings and ring homomorphisms
- 1.2 Ideals, quotient rings
- 1.3 Zero divisors, nilpotent elements, units
- 1.4 Prime ideals and maximal ideals
- 1.5 Nilradical and Jacobson radical
- 1.6 Operations on ideals
- 1.7 Ideal quotient, annihilator, and radical
- 1.8 Extension and contraction

Chap. 2. Modules

- 2.1 Modules and module morphisms
- 2.2 Submodules and quotient modules
- 2.3 Operations on submodules
- 2.4 Direct sum and product
- 2.5 Finitely generated modules
- 2.6 Exact sequences
- 2.7 Tensor product of modules
- 2.8 Restriction and extension of scalars
- 2.9 Exactness properties of the tensor product
- 2.10 Tensor product of algebras

Chap. 3. Rings and Modules of Fractions

- 3.1 Localization of rings
- 3.2 Localization of modules
- 3.3 Local properties
- 3.4 Extended and contracted ideals in localizations
- 3.5 Spectrum of a ring and Zariski topology
- 3.3 Pull back of ring morphisms

Chap. 4. Integral Dependence and Valuations

- 3.1 Integral dependence
- 3.2 Going-up theorem
- 3.3 Integrally closed domains: going down theorem
- 3.4 Noether normalization lemma and Hilbert's Nullstellensatz
- 3.5 Valuations rings

Chap. 5. Chain Conditions

Chap. 6. Noetherian Rings

Chap. 7. Primary Decomposition

Chap. 8. Dimension Theory

- 8.1 Graded rings and modules
- 8.2 Hilbert functions
- 8.3 Hilbert-Samuel function of local rings
- 8.4 Dimension theory of local rings

Main reference

Introduction to commutative algebra
M. F. Atiyah ; I. G. Macdonald., Addison-Wesley 2018.