Questions for the talks given by Jim Orlin.

1. Which of the following claims are true and which are false? Justify your answer either by giving a brief justification or by constructing a counterexample.

1. If *x* is a maximum flow, either *xij* = 0 or *xji* = 0 for every arc (*i, j*) ∈ *A*.
2. Any network always has a maximum flow *x* such that *xij* = 0 or *xji* = 0 for all (*i*, *j*) ∈ *A*.
3. If we multiply each arc capacity by the same positive integer *K*, the minimum cut(s) remain unchanged.
4. If we add the same positive integer *K* to each arc capacity, the minimum cut(s) remain unchanged.

2. Let *G* = (*N*, *A*) be a network for the max flow problem, and let (*i*, *j*) be some arc of *A*.

1. Suppose that *xij* = *uij* in every maximum flow. Show that *xij* = *uij* in some minimum *s*-*t* cut.
2. Show by example that it is possible that that *xij = uij* in every max flow and there is some minimum *s-t* cut that does not contain (*i*, *j*)
3. Suppose that *xij* ≠ *uij* in some maximum flow. Show that (*i*, *j*) cannot be in any minimum *s*-*t* cut.

3. Show that the number of non-saturating pushes in the wave and excess scaling algorithm is O(n2 log½ U + n log U), where *U* is the largest capacity in the network. (Assume that *U* > 2). Assume also that log U < n, and so n log U < n2. We divide this problem into several parts. For each part give a brief explanation. (Your explanation should identify the key insights that can be turned into a rigorous proof.) As in the lecture, let Θ = ∑i active *e*(*i*)d(*i*)/∆ be the potential function of the algorithm at the ∆-scaling phase. Recall that an active node always has a distance label less than n. As in the lecture, a phase ends immediately after there is a wave with fewer than *n*/ log½ *U* relabels. At the end of a phase, ∆ is replaced by ∆/2. (A small push has flow less than ∆/2. A large push as at least ∆/2 units of flow.)

Facts you may use.

1. The number of scaling phases is O(log U).
2. Any node that was not relabeled during a wave has an excess of 0 at the end of the wave.
3. Show that all waves in a scaling phase (except possibly the last wave) result in at least n/ log½ U nodes being relabeled. Show that the total number of waves over all scaling phases is O((n log½ U) + log U).
4. Show that the number of small non-saturating pushes during a wave is O(n), and thus the number of small non-saturating pushes over all waves is   
   O(n2 log½ U + n log U).
5. Show that the total excess at the end of the ∆-scaling phase is O(n∆/ log½ U).
6. Show that the increases in potential function due to the changes in the scaling parameter is O(n2 log½ U) over all scaling iterations.
7. Use a potential function argument to show that the number of large pushes over all scaling phases is O(n2 log½ U).