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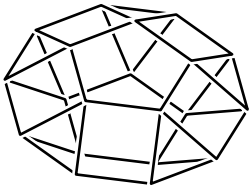
SoSe 2016
21.04.2016

Discrete Geometry II
Tutorial Sheet 1

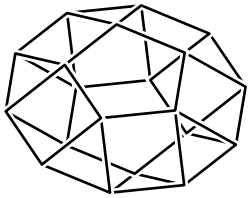
All the information about the lecture, tutorials and literature will be available on the homepage of this course:

`www3.math.tu-berlin.de/combi/dmg/teaching/SoSe16/VL-Discrete+Geometry+II.html`

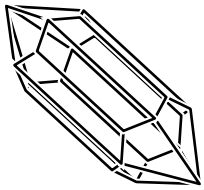
Exercise T1 (planar nets)
Assign the polytopes to their planar nets (see also next page).



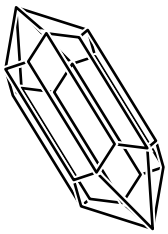
(a) Pentagonal orthobicupola



(b) Pentagonal gyrobicupola



(c)

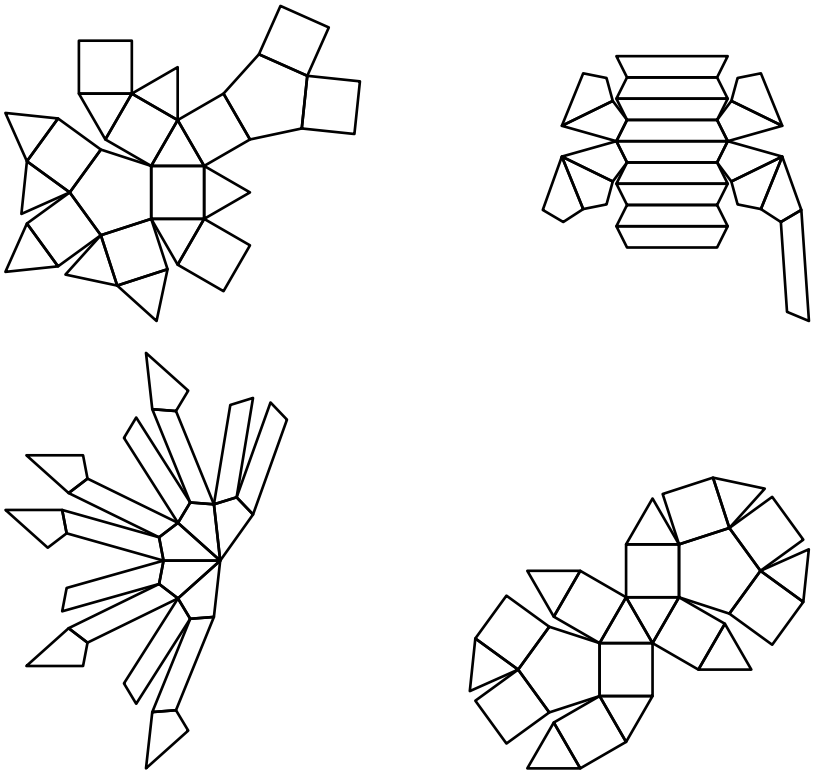


(d)

Exercise T2 (3-polytopes) (5 points)
Let P be a simple 3-dimensional polytope, whose facets are either pentagons or hexagons. How many pentagonal facets does the polytope P have? Give at least two distinct examples for such polytopes.

Exercise T3 (f-vectors of polytopes) (5 points)
(a) The product of two polytopes $P \subset \mathbb{R}^d$ and $Q \subset \mathbb{R}^e$ is

$$P \times Q = \{(v, w) \in \mathbb{R}^{d+e} \mid v \in P \text{ and } w \in Q\}$$



Give a formula for the f-vector of $P \times Q$.

(b) Let $P \subset \mathbb{R}^d$ be a polytope with 0 in the relative interior. The bipyramid B over P is the convex hull

$$B \;=\; \operatorname{conv}\Big(\{(v,0) \in \mathbb{R}^{d+1} \mid v \in P\} \cup \{(0,1),(0,-1)\}\Big)$$

What is the f-vector of the bipyramid B?

Exercise T4 (linear programming) (5 points)
Consider the Maximum Bipartite Matching Problem: Given a finite bipartite graph $G = (A \dot\cup B, E)$, find a set $S \subseteq E$ of edges such that no pair of edges in S share a common node and the cardinality $\#S$ is maximal.

(a) Model this problem as an *integer linear program*.

(b) Dualize the LP-relaxation of that problem.

(c) What does an optimal solution of the dual problem describe in terms of G?