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Discrete Geometry II
Tutorial Sheet 2

Exercise T1 (face figures) (5 points)
Let P be the n-dimensional cross polytope and let F be an arbitary dimensional face of P. Describe the face lattice of the face figure $\mathcal{F}(F, P)$ and its f-vector.

Exercise T2 (polarity) (5 points)
Let $P \subset \mathbb{R}^d$ and $Q \subset \mathbb{R}^e$ be full-dimensional polytopes, both with the origin in the interior.

a) Describe $(P^\circ \times Q^\circ)^\circ$ in the case that P is the intervall $[-1, 1]$ and Q is a regular n-gon centered at 0.

b) In the general case, describe the vertices of $(P^\circ \times Q^\circ)^\circ$ in terms of the vertices of the polytopes P and Q.

Exercise T3 (Gale's evenness condition) (5 points)
The moment curve is the image of the map $\mu_d : \mathbb{R} \to \mathbb{R}^d$ with $\mu_d(t) = (t, \ldots, t^d)$. Let $t_1 < t_2 < \ldots < t_n$ be real numbers and $I \subset [n]$ be a index set of size d. The convex hull of $\mu_d(t_1)$, $\mu_d(t_2), \ldots,$ $\mu_d(t_n)$ is a cyclic polytope $Z_d(n)$.
Prove that the convex hull of $\{\mu_d(t_i) \mid i \in I\}$ forms a facet of $Z_d(n)$ if and only if for each pair $i, j \notin I$ the size of $\{k \in I \mid i < k < j\}$ is even.