Technische Universität Berlin
Department of Mathematics
Prof. Michael Joswig
Benjamin Schröter

SoSe 2016
19.05.2016

Discrete Geometry II
Tutorial Sheet 4

Exercise T1 (planar convex hull complexity) (4 points)
Consider a convex hull algorithm that takes points $v_1, v_2, \ldots, v_m$ in $\mathbb{R}^2$ and returns the vertices of $\operatorname{conv}\{v_1, v_2, \ldots, v_m\}$ in cyclic order.
Reduce the problem of sorting (distinct) real numbers $b_1, b_2, \ldots, b_m \in \mathbb{R}$ to the above algorithm, i.e., find an efficient procedure to sort the numbers that uses the convex hull algorithm.
What is the run time of your procedure and what is the run time of a sorting algorithm under the assumtion that the algorithem uses pairwise comparision? What does this reduction imply for the complexity for convex hull algorithms?

Exercise T2 (Voronoi diagrams) (4 points)
Let S be a finite point set.

(a) Give conditions which imply that all Voronoi regions of S are pointed polyhedra.
(b) Which Voronoi regions of S are unbounded?

Exercise T3 (triangulations) (7 points)
Consider the affine halfspace

$$H_d^+ = \left\{ x \in \mathbb{R}^d \mid \sum_{i=1}^{d} x_i \leq \frac{3}{2} \right\} .$$

Intersect the cube $[0,1]^d$ with H_d^+ and translate the polytope by the vector $(-1/d, \ldots, -1/d) \in \mathbb{R}^d$. This polytope is the dwarfed cube D. Let v be the vertex in the polar polytope D° that corresponds to the facet defined by the traslation of H_d^+.

(a) Show that D° is full-dimensional and the origin is in its interior.
(b) How many vertices does D have?
(c) Consider a placing triangulation of D° obtained by the Beneath-and-Beyond algorithm, where the vertex v is placed as last vertex. Show that in this triangulation the number of d-simplices which contain v is $2^d - d - 1$.