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Discrete Geometry II
Tutorial Sheet 5

Exercise T1 (Convergence) (5 points)
Let $S \subset \mathbb{R}^2$ be finite and $(s_1, s_2) \in S$. The goal is to show

$$\lim_{\varepsilon \to 0^+} \operatorname{Par}(s, H_{s_2-\varepsilon}) = \left\{ \begin{pmatrix} s_1 \\ \sigma \end{pmatrix} \in \mathbb{R}^2 \mid \sigma \geq s_2 \right\} .$$

Where the limit can be interpreted in different ways.

(a) As first step give an explicit parametrization for $\operatorname{Par}(s, H_{s_2-\varepsilon})$.

(b) Show pointwise convergence, i.e., find for each $\delta > 0$ and $y \geq s_2$ parameters $\varepsilon > 0$ and $x \in \mathbb{R}$ such that $(x, y) \in \operatorname{Par}(s, H_{s_2-\varepsilon})$ and the (Euclidian) distance of (x, y) and (s_1, y) is smaller than δ.

(c) Let $\mathbb{B}(0, r) = \{(x_1, x_2) \in \mathbb{R}^2 \mid \max\{|x_1|, |x_2| \leq r\}\}$ the closed ball centered at the orgin with radius r in the maximum norm. Pick $r > x_1^2 + x_2^2$ for each $(x_1, x_2) \in S$. Together with the Euclidian distance this is a metric space. Consider

$$\operatorname{Par}(s, H_{s_2-\varepsilon}) \cap \mathbb{B}(0, r) \text{ and } \left\{ \begin{pmatrix} s_1 \\ \sigma \end{pmatrix} \in \mathbb{R}^2 \mid \sigma \geq s_2 \right\} \cap \mathbb{B}(0, r) .$$

Show that these sets are compact in $\mathbb{B}(0, r)$. Show convergence w.r.t. the Hausdorff metric in the space of non-empty and compact subsets of $\mathbb{B}(0, r)$.

Exercise T2 (Beach line algorithm) (5 points)
Let $S \subset \mathbb{R}^2$ be a finite point set. Show that for each generic $\tau \in \mathbb{R}$ there are at most $2|S| - 2$ breakpoints in the beach line B_τ.

Exercise T3 (infinite Voronoi diagrams) (5 points)
Let $S \subset \mathbb{R}^d$ be an infinite set such that for each $r > 0$ the set $\mathbb{B}(0, r) \cap S$ is finite. We define the Voronoi region $\operatorname{VR}_S(s) = \{x \in \mathbb{R}^d \mid \operatorname{dist}(x, s) \leq \operatorname{dist}(x, t) \text{ for all } t \in S\}$ as in the finite case.

(a) Show that the Voronoi regions generate an infinite complex of $\mathbb{R}^d$.

(b) Let S be the face centered cubic lattice

$$\mathbb{Z}\begin{pmatrix}2\\0\\0\end{pmatrix}+\mathbb{Z}\begin{pmatrix}1\\1\\0\end{pmatrix}+\mathbb{Z}\begin{pmatrix}1\\0\\1\end{pmatrix}.$$

Describe the Voronoi regions $\mathrm{VR}_S(s)$ for all $s \in S$.