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Discrete Geometry II
Tutorial Sheet 6

Exercise T1 (Delone triangulations) (5 points)
Show that if every $(n+2)$-element subset of $S \subset \mathbb{R}^n$ does not lie on a common $(n-1)$-sphere, then the lifted polyhedron $\mathcal{P}(S)$ is simple and therefore every Voronoi region is simple. Deduce that the Delone subdivision $\mathrm{DS}(S)$ of S is a triangulation.

Exercise T2 (projective transformation to a polytope) (Bonus: 5 points)
Let $S \subset \mathbb{R}^n$ be a finite point set, π be the projective transformation introduced in the lecture and $\mathcal{P}(S) \subset \mathbb{R}^{n+1}$ the polyhedron whose vertical projection is the Voronoi diagram $\mathrm{VD}(S)$. Show that the closure of the image $P_S = \overline{\pi(\mathcal{P}(S))}$ is a polytope, under the assumtion that $0 \in S$ and for all $x \in \mathrm{VR}_S(0)$ holds $||x|| \leq 1$.

Exercise T3 (Voronoi diagrams w.r.t the Chebyshev distance) (5 points)
Draw the Voronoi diagram of

$$S \;=\; \left\{ \begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix} \begin{pmatrix}-1\\3\end{pmatrix}, \begin{pmatrix}-3\\-3\end{pmatrix}, \begin{pmatrix}3\\-2\end{pmatrix}, \begin{pmatrix}0\\-4\end{pmatrix}, \right\} \;.$$

w.r.t the Chebyshev distance also known as maximum metric. Show that this Voronoi diagram is not the orthogonal projection of the boundary complex of a polytope.

Exercise T4 (random points) (Bonus: 10 points)
Write a `polymake`/`perl` script (or a `C++` client) that computes a Floating-point-approximation of the volume of a not-necessarily full-dimensional simplex. Extend your script/client such that it computes the volume of an arbitrary bounded polytope.
Please hand in your code via e-mail at `schroeter@math. tu-berlin. de` *not later than June 15. A template will be available: at the homepage of the course.*