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## Discrete Geometry II

### Tutorial Sheet 7

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**Exercise T1** (simplices in a Delone triangulation) (5 points)

Let  $\mathcal{D}$  be a Delone triangulation of a point set  $S$ . The vertices of a full-dimensional simplex  $\Delta \in \mathcal{D}$  lie on a unique sphere with center  $c$  and radius  $\rho$ . Show that for any two simplices  $\Delta, \Delta'$  in  $\mathcal{D}$  that contain  $x$  we have

$$\psi_{\mathcal{D}}(x, \Delta) = \rho^2 - \|x - c\|^2 = (\rho')^2 - \|x - c'\|^2 = \psi_{\mathcal{D}}(x, \Delta') .$$

**Exercise T2** (A classical proposition from Euclid's 'The Elements') (5 points)

Show that in a quadrangle  $ABCD$  in  $\mathbb{R}^2$  the angle  $ADB$  equals the angle  $ACB$  if and only if  $A, B, C$  and  $D$  lie on a common circle. Furthermore show that the angle  $ADB$  is larger than  $ACB$  if and only if  $D$  is on the inside of the circle through  $A, B$  and  $C$ .

**Exercise T3** (Counting all triangulations of an  $n$ -gon) (5 points)

Show that the number of triangulations of an  $n$ -gon without additional vertices is the Catalan number

$$C_{n-2} = \frac{1}{n-1} \binom{2n-4}{n-2} .$$

*Hint: Find a suitable recursion.*