Technische Universität Berlin
Department of Mathematics
Prof. Michael Joswig
Benjamin Schröter

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Discrete Geometry II
Tutorial Sheet 8

Exercise T1 (Unique intersection point) (5 points)
Let $r < 1$, C be a Jordan curve, $s^{(1)}$, $s^{(2)}$ be two neigboring r-sample points on C and p an intersection point of the bisector of the line segment $[s^{(1)}, s^{(2)}]$ and the curve arc between $s^{(1)}$, $s^{(2)}$. Show that the point p is unique.

Exercise T2 (Angles between sample points) (5 points)
Show that for three neighboring sample points $s^{(1)}$, $s^{(2)}, s^{(3)}$ on a Jordan curve, the (smaller) angle between the segments $[s^{(1)}, s^{(2)}]$ and $[s^{(2)}, s^{(3)}]$ is at least $\pi - 2\arcsin(r/2)$ for $r < 2$. In particular this angle is larger than $\frac{\pi}{2}$ for $r \leq \frac{1}{3}$

Exercise T3 (NN-Crust) (5 points)

Input: finite sample $S \subset \mathbb{R}^2$
Output: a subset $G(S)$ of edges of the Delone subdivision of S
Compute the edge set D of the Delone subdivision of S.
Compute the edge set $N \subset D$ which connects nearest neighbors in S.
$G \leftarrow N$
foreach *sample point* $x \in S$ *that is contained in exactly one edge* $e \in N$ **do**
  Determine the shortest edge $e' \in D$ containing s that forms an angle greater that $\pi/2$ with e.
  $G \leftarrow G \cup \{e'\}$
end
return G

Consider the above `NN-Crust` algorithm with m sample points. Provide an exact formulation of steps 2 to 5 which have in total a cost of at most $O(m)$.