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Discrete Geometry II
Tutorial Sheet 9

Exercise T1 (Curve reconstruction) (5(+2) points)

a) Give an example of a smooth curve and r sample points on that curve, such that the `NN-Crust` algorithm returns the correct neighbouring relations between the sample points.

(Bonus: Determine a reasonable bound for the parameter r.)

b) Give an explicit example of a curve and r sample points, such that the graph that `NN-Crust` algorithm returns has at least one node of degree 1 and one node of degree higher or equal to 3.

Exercise T2 (spline interpolation) (10 points)

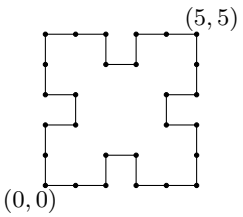
Recall a cubic spline interpolation of the points $x_0, \dots x_{n-1} \in \mathbb{R}^2$ is defined by the closed curve $C : [0, n] \to \mathbb{R}^2$ with the piecewise parametrization

$$C(t) \;=\; p_i(t) \;:=\; (i-t)^3 a_i + (i-t)^2 b_i + (i-t)c_i + d_i \text{ for } t \in [i, i+1)$$

and $C(n) = C(0)$, such that this parametrization has the following properties for $k \in \{0, 1, 2\}$ and $i \in \{0, \dots, n-1\}$

$$\begin{aligned} C(i) &= x_i \\ \frac{\mathrm{d}^k}{\mathrm{d}t^k} C(n) &= \frac{\mathrm{d}^k}{\mathrm{d}t^k} C(0) \\ \frac{\mathrm{d}^k}{\mathrm{d}t^k} C\,(i+1) &= \frac{\mathrm{d}^k}{\mathrm{d}t^k} p_i\,(i+1) \end{aligned}$$

Determine the coefficents $a_0, \dots, d_{n-1}$ for the spline interpolation that gives you a curve throug the points in the following picture.



(The calculation for the coefficents is not required.) Compare your curve with the curve that the `LaTeX` package `TiKz` produces.