Exercise T1 (Curve reconstruction) (5(+2) points)

a) Give an example of a smooth curve and \( r \) sample points on that curve, such that the \texttt{NN-Crust} algorithm returns the correct neighbouring relations between the sample points.

(Bonus: Determine a reasonable bound for the parameter \( r \).)

b) Give an explicit example of a curve and \( r \) sample points, such that the graph that \texttt{NN-Crust} algorithm returns has at least one node of degree 1 and one node of degree higher or equal to 3.

Exercise T2 (spline interpolation) (10 points)

Recall a cubic spline interpolation of the points \( x_0, \ldots, x_{n-1} \in \mathbb{R}^2 \) is defined by the closed curve \( C : [0, n] \to \mathbb{R}^2 \) with the piecewise parametrization

\[
C(t) = p_i(t) := (i-t)^3a_i + (i-t)^2b_i + (i-t)c_i + d_i \quad \text{for} \ t \in [i, i+1)
\]

and \( C(n) = C(0) \), such that this parametrization has the following properties for \( k \in \{0, 1, 2\} \) and \( i \in \{0, \ldots, n-1\} \)

\[
\frac{d^k}{dt^k} C(i) = x_i \\
\frac{d^k}{dt^k} C(n) = \frac{d^k}{dt^k} C(0) \\
\frac{d^k}{dt^k} C(i+1) = \frac{d^k}{dt^k} p_i(i+1)
\]

Determine the coefficients \( a_0, \ldots, d_{n-1} \) for the spline interpolation that gives you a curve through the points in the following picture.
(The calculation for the coefficients is not required.) Compare your curve with the curve that the \LaTeX{} package TiKz produces.