
Discrete Geometry II

Tutorial Sheet 10

Exercise T1 (The flip graph of the 3-cube) (5 points)

A subdivision T of a point configuration is *almost a triangulation* if it is not a triangulation but all its proper refinements are triangulations. Two triangulations of the same point configuration are connected via a *flip supported on the almost triangulation* T if they are the only two triangulations refining T .

Describe all triangulations and the flip-graph of the prism over a triangle.

Exercise T2 (Catalan numbers) (10 points)

The Catalan numbers C_n count many things for example triangulations of an $(n+2)$ -gon. Show that the following sets have also cardinality C_n .

- i) Ways of connecting $2n$ points in the plane lying on a horizontal line by n nonintersecting arcs, each arc connects two points and lying above the points.
- ii) Sequences a_1, \dots, a_n of integers such that $a_1 = 0$ and $0 \leq a_{i+1} \leq a_i + 1$.
- iii) 312 – *avoiding* permutations of $[n]$, i.e., there does not exist $i < j < k$ such that in permutation $a_1 a_2 \dots a_n$ we have $a_j < a_k < a_i$.
- iv) Noncrossing partitions of $[n]$, i.e., partitions $\{A_1, \dots, A_k\}$ such that if $a < b < c < d$ and $a, c \in A_i$ and $b, d \in A_j$, then $i = j$.

Find at least one other family of sets whose elements have the cardinality C_n .