Exercise 1.1 *Show that for* $1 \leq p < \infty$ *the space* l_p *is separable, i.e., it contains a countable dense subset. What about* l_∞*?*

Exercise 1.2 *Let* $1 \leq p \leq 2$ *and* $x, y \in \mathbb{R}$. *Show that*

$$|x-y|^p + |x+y|^p \leq 2|x|^p + 2|y|^p.$$

Exercise 1.3 *Let* $1 \leq p \leq 2$. *Infinitely many balls* $B_p(\mathbf{z}, r)$ *can be packed into the unit ball* $B_p(\mathbf{0}, 1)$ *if and only if*

$$r \leq \left[1 + 2^{1-1/p}\right]^{-1}.$$

For $1 \geq r > \left[1 + 2^{1-1/p}\right]^{-1}$ *let* $m_p(r)$ *be the maximum number of* l_p*-balls* $B(\mathbf{z}, r)$ *which can be packed into the unit* l_p*-ball* $B(\mathbf{0}, 1)$. *Then*

$$m_p(r) \leq 1 + \left(\frac{1-r}{r}\right)^p \left(\frac{1 - 2^{1-p}}{1 - 2^{1-p}\left(\frac{1-r}{r}\right)^p}\right).$$

Exercise 1.4 *Let* $x \geq y \in \mathbb{R}_{\geq 0}$. *Show that*

$$x^p - y^p \begin{cases} \leq (x-y)^p, & 0 \leq p \leq 1, \\ \geq (x-y)^p, & 1 \leq p < \infty. \end{cases}$$

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