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Convex Geometry I

WS 2016

Exercise sheet 1

Discussion: Friday, 28.10.2016.

Exercise 1.1 *Prove the following:*

i) $\mathbf{x}_1, \ldots, \mathbf{x}_m \in \mathbb{R}^n$ *are affinely independent if and only if for every* $k \in \{1, \ldots, m\}$ *the points* $\mathbf{x}_i - \mathbf{x}_k$, $i = 1, \ldots, k-1, k+1, \ldots, m$, *are linearly independent.*

ii) *If* $A \subseteq \mathbb{R}^n$ *is a non-empty affine subspace, then there is a linear subspace* U *of* $\mathbb{R}^n$ *and a point* $\mathbf{v} \in A$, *such that* $A = \mathbf{v} + U$. *The dimension of such a set* A *will be defined as* $\dim(A) := \dim(U)$. *In addition one sets* $\dim(\emptyset) = -1$.

Let $X \subseteq \mathbb{R}^n$ *be non-empty.*

iii) *For every* $\mathbf{v} \in X$ *we have* $\operatorname{aff} X = \mathbf{v} + \operatorname{lin}(X - \mathbf{v})$ *and therefore* $\dim X = \dim \operatorname{lin}(X - \mathbf{v})$.

iv) $\dim X + 1$ *is the maximal number of affinely independent points in* X.

Exercise 1.2 *Prove or disprove.*

i) *If* $X \subseteq \mathbb{R}^n$ *is closed, then also* $\operatorname{conv} X$ *is closed.*

ii) *If* $X \subseteq \mathbb{R}^n$ *is convex, then the closure of* X *is convex.*

iii) *If* $X \subseteq \mathbb{R}^n$ *is open, then* $\operatorname{conv} X$ *is open.*

iv) *If* $X \subseteq \mathbb{R}^n$ *is compact, then* $\operatorname{conv} X$ *is compact.*

Exercise 1.3 *Let* $X_1, X_2 \subseteq \mathbb{R}^n$. *Show that*

$$\operatorname{conv}(X_1 + X_2) = \operatorname{conv} X_1 + \operatorname{conv} X_2.$$

Is it also true for the positive hull $\operatorname{pos}()$ *instead of* $\operatorname{conv}()$*?*

Exercise 1.4

i) *Let* $C \in \mathcal{C}^n$ *and let* $\mathcal{X}$ *be a finite family of convex sets in* $\mathbb{R}^n$, *such that for each choice* $X_1, \ldots, X_{n+1} \in \mathcal{X}$ *there is a translate of* C, *that is contained in* $\cap_{i=1}^{n+1} X_i$. *Show, that there is a translate of* C *that is contained in* $\cap_{X \in \mathcal{X}} X$.

ii) *Let* $l_1, \ldots, l_m \subset \mathbb{R}^2$, $m \geq 3$, *be a collection of parallel line segments. Show, that if for any three of the segments there is a line passing through them, there is a line passing through all segments.*