TECHNISCHE UNIVERSITÄT BERLIN SS07
Fakultät II - Institut für Mathematik
John Sullivan / Charles Gunn Abgabe: 6.6.7

6. Übung Visualisierung in der Mathematik

(Line equations and bouncing balls)

Übungsaufgaben

Information: The course web site is http://www.math.tu-berlin.de/geometrie/Lehre/SS07/MathVis/. All assignments will be posted there in .pdf format. You can also check the web site for contact information for the teachers, and other information related to this class.

1. Aufgabe

Line equations and line coordinates

- Suppose $P = (x_0, y_0, 1)$ and $Q = (x_1, y_1, 1)$ are points in R^2 in homogeneous coordinates. Show that the line equation for the line joining P and Q is given by $ax+by+c = 0$ where $[a, b, c] = P \times Q$ where $\times$ is the cross product [deutsch: Vektorprodukt] of the two points. We call $[a, b, c]$ the *line coordinates* of the line.
- For a point $P = (x, y, 1)$ and line $l = [a, b, c]$ define $\langle P, l\rangle := ax + by + c$. Then $\langle P, l\rangle = 0 \iff P \in l$, so $\langle , \rangle$ is an *incidence operator* which is 0 exactly when the point P lies on the line l. Suppose l is defined as above by the two points P and Q and that $S = (x_2, y_2, 1)$ is third point that doesn't lie on l, that is, $d := \langle S, l\rangle \neq 0$. Imagine you are positioned on line l at P facing point Q. Show that $d > 0 \iff S$ lies on your left side.
- Deduce that $\langle S, l\rangle > 0 \iff$ the points P, Q, S lie in counter-clockwise order [deutsch: Gegenuhrzeigersinn] on $\triangle OPQ$.

2. Aufgabe

Reflection in a line Let $P = (x_0, y_0, 1)$ be a point and $V = (x_v, y_v, 0)$ be a vector, both in R^2. Let $l = [a, b, c]$ be a line. Recall that the reflection in the line l is given by the 3x3 matrix:
$R_l = \begin{pmatrix} 1-2a^2 & -2ab & -2ac \\ -2ab & 1-2b^2 & -2bc \\ 0 & 0 & 1 \end{pmatrix}$ (Important: this formula assumes $a^2+b^2 = 1$. If that isn't the case, be sure to normalize the line coordinates to make it so!) Hence, the reflection of P in the line l is given by the product $R_l(P)$. Is it also true that the reflection of V in the line l is given by the product $R_l(V)$? Justify your reasoning.

3. Aufgabe

Convex polygons Assume we have a convex polygon $\mathbb{P}$ in the Euclidean plane given by the n points P_i, $1 \leq i \leq n$. Assume these points are represented in homogeneous coordinates by $P_i = (x_i, y_i, 1)$. Define a set of n lines l_i by $l_i := \overleftrightarrow{P_i P_j}$ where $j = (i+1) \mod n$. Then $\{l_i\}$ are the set of bounding lines of $\mathbb{P}$. Represent these lines by their line coordinates as defined above. Define the center of gravity C of $\mathbb{P}$ to be the average of the points P_i. Show:

- The numbers $\{\langle C, l_i\rangle\}$ all have the same sign. Define $\epsilon := 1$ if they are all positive, and $\epsilon := -1$ if they are all negative.
- $\epsilon = 1 \iff \mathbb{P}$ has a counterclockwise orientation.
- For a variable point $P = (x, y, 1)$, $\epsilon\langle P, l_i\rangle > 0 \ \forall i \iff P \in \mathbb{P}$. [Hint: a point lies within $\mathbb{P}$ exactly when it lies on the same side of each bounding line, as C does.]