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http://www3.math.tu-berlin.de/geometrie/Lehre/SS12/ComplexAnalysis SS 12

Complex Analysis - Exercise Sheet 11

- compact convergence, Riemann mapping theorem -

due 4-5.07.2012

Tutorial

Exercise 1

Let $G \subset \mathbb{C}$ be a region and $(f_n)_{n\in\mathbb{N}}$ a sequence of holomorphic functions. Let further F_n be an anti–derivative of f_n. Show that the sequence of anti–derivatives $(F_n)_{n\in\mathbb{N}}$ converges compactly in G if and only of the sequence $(f_n)_{n\in\mathbb{N}}$ converges compactly and there is a point $c \in G$ such that $(F_n(c))_{n\in\mathbb{N}}$ converges.

Exercise 2

Let $0 \leq \alpha < \beta \leq 2\pi$ and define $S_{\alpha,\beta} := \{re^{\varphi} \mid 0 < r,\, \alpha < \varphi < \beta\}$. Find a biholomorphic map between any two $S_{\alpha,\beta}$. Denote further by $C^\sharp$ the interior of the intersection of a circle C and a half plane, such that $C^\sharp \neq \emptyset$ and $C^\sharp \neq C$. Find a biholomorphic map between $C^\sharp$ and $S_{\alpha,\beta}$.

Exercise 3

From Liouville's theorem follows that the complex plane and the unit disk $\mathbb{D}$ are not biholomorphic equivalent. Is there any other region than the plane itself it is biholomorphic equivalent to? Are there other regions than the complex plane which are not biholomorphic equivalent to a region in $\mathbb{D}$?

Homework

Exercise 1 **6 points**

Let G be a bounded region and $f\colon G \to G$ a holomorphic mapping. Let z_0 be a fixed point of f where $\left|f'\big(z_0\big)\right| < 1$. Consider

$$f_n := \underbrace{f \circ \cdots \circ f}_{n-times}$$

the n–th iteration of f. Show that f_n converges compactly to $f(z) = z_0$.

Exercise 2 **8 points**

Let $\mathbb{D} := \{z \in \mathbb{C} \mid |z| < 1\}$ and $Q := \{z \in \mathbb{C} \mid \operatorname{Re} z > 0, \operatorname{Im} z > 0\}$ denote the unit disk and the first quadrant, resp. Find a biholomorphic mapping between

1. $\mathbb{D}$ and $\mathbb{C} \setminus \{x \geq 0\}$,
2. Q and $\mathbb{D}$,
3. Q and $\mathbb{D} \bigcap \big\{z \in \mathbb{C} \mid \operatorname{Im} z > 0\big\}$,
4. Q and $\mathbb{D} \bigcap Q$.

Exercise 3 **6 points**

Are $\mathbb{C} \setminus \big\{\big[-1, 1\big]\big\}$ and $\mathbb{D} \setminus \{0\}$ biholomorphic equivalent?