TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik
Prof. Dr. John Sullivan
Isabella Thiesen

Differential Geometry I - SS 15

http://www3.math.tu-berlin.de/geometrie/Lehre/SS15/DGI/

Homework 5

Problem 1 (2+3 Points)

(a) Show that the hyperbolic paraboloid $S = \{(x, y, z) \in \mathbb{R}^3 : z = xy\}$ is a doubly ruled surface, i.e., that it is swept out by two distinct families of straight lines. Hint: Show that S can be parameterized as a ruled surface in two essentially different ways.

(b) A generalized cone is a ruled surface of the form $\mathbf{x}(t, u) = c + u\delta(t)$ and a generalized cylinder is a ruled surface of the form $\mathbf{x}(t, u) = \beta(t) + uc$, where in both cases $c \in \mathbb{R}^3$ is a constant. Sketch both types of surface and check under which conditions (and for which (t, u)) the derivative $d\mathbf{x}$ is injective.

Problem 2 (3+2 Points)

Let $\alpha(s) = (r(s), 0, z(s))$ be a unit-speed curve in the xz-plane with $r > 0$. Consider the corresponding surface of revolution $\mathbf{x}(s, \theta) = (r(s)\cos\theta, r(s)\sin\theta, z(s))$ and compute

(a) its normal vector and first and second fundamental form,

(b) its principal curvatures and principal curvature directions.

Check your formulas in the case $\alpha(s) = (\cos s, 0, \sin s)$ for $s \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Problem 3 (3 Points)

The unit three-dimensional sphere $\mathbb{S}^3 \subset \mathbb{R}^4$ can be parametrized by

$$f(\phi, \psi, \theta) = (\cos\phi\cos\psi\cos\theta, \sin\phi\cos\psi\cos\theta, \sin\psi\cos\theta, \sin\theta).$$

For which values of (ϕ, ψ, θ) is df injective?

Problem 4 (3 Points)

The *Mercator projection*

$$\mathbf{x}(u, \phi) = \frac{1}{\cosh u}\big(\cos\phi, \sin\phi, \sinh u), \quad 0 < \phi < 2\pi, u \in \mathbb{R}$$

is a parametrization of the unit sphere excluding the north and south pole. Show that this parametrization is conformal (angle-preserving).

Homework due in at the start of lecture on Thursday 28 May.