TECHNISCHE UNIVERSITÄT BERLIN
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Geometry II - SS 16
Discrete Differential Geometry

http://www.math.tu-berlin.de/geometrie/Lehre/SS16/GeometryII/

Exercise Sheet 10

Exercise 1: Discrete Gauss curvature. (4 pts)

(a) Show that a convex polyhedron has positive Gauss curvature at every vertex.

(b) Give an example of a non-convex polyhedron with positive Gauss curvature at every vertex to show that the converse is not true.

Exercise 2: Simplical faces and simple vertices of a convex polyhedron. (4 pts)

The degree $\deg(v)$ of a vertex v of a discrete surface is the number of edges adjacent to it. The degree $\deg(f)$ of a face f of a discrete surface is the number of edges its boundary consists of.

Show that every convex polyhedron has a triangle face or a vertex of degree 3 (or both). Even stronger: Show that the number of triangle faces plus the number of vertices of degree 3 is at least eight, so there are at least four triangle faces or four vertices of degree 3.

Hint: Use the double counting formula:

$$\sum_{v \in V} \deg(v) = \sum_{f \in F} \deg(f) = 2|E|$$

and the fact that a convex polygon is topologically a sphere.

Exercise 3: Straight curves on a polyhedral surface. (5 pts)

Let S be a polyhedral surface with vertices V. For each point $P \in S \setminus V$, there is a neighbourhood which is isometric to an open disc in $\mathbb{R}^2$. (For points in the interior of a 2-face this is trivial, for points in the interior of an edge one just has to "unfold" along the edge). Let γ be a curve in $S \setminus V$. The curve γ is called *straight*, if all images of γ under the above described isometries are straight line segments.

(a) Express in terms of angles how a straight curve γ crosses the edges of S.

(b) Find a polyhedral surface S which contains two points P and Q, such that there are infinitely many straight lines between P and Q (with different traces).

PLEASE TURN!!!

Exercise 4: Equihedral tetrahedra. (5 pts)

Let $\Delta \subseteq \mathbb{R}^3$ be a tetrahedron. Prove that the following conditions are equivalent:

(a) All faces of Δ are congruent triangles

(b) All faces of Δ have equal perimeter

(c) All vertices of Δ have equal curvature

Such tetrahedra are called *equihedral tetrahedra*.

Exercise 5: Paraboloid of revolution. (6 pts)

The set $(x, y, z) \in \mathbb{R}^3 | z = x^2 + y^2$ describes a paraboloid of revolution.

(a) Find a parametrization of this surface

(b) Use your parametrization to determine the tangent plane at a point

(c) Determine the Gauss map and its image (a set in $\mathbf{S}^2$).

(d) Determine the Gauss and mean curvature.

Due: Before tutorials on July 13/14, 2016.