TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry II - SS 16
Discrete Differential Geometry

http://www.math.tu-berlin.de/geometrie/Lehre/SS16/GeometryII/

Exercise Sheet 8

Exercise 1: Darboux transforms and signed area. (5 pts)

Let $\gamma : \{0, \dots n\} \to \mathbb{R}^2$ be a discrete planar curve and let $p \in \mathbb{R}^2$ be any point. The total signed area of the triangles $(p, \gamma_k, \gamma_{k+1})$ is

$$A(p, \gamma) = \frac{1}{2} \sum_{k=0}^{n-1} \det \begin{pmatrix} \gamma_k - p & \gamma_{k+1} - p \end{pmatrix}.$$

Assuming that γ is a closed curve, show that $A(p, \gamma) =: A(\gamma)$ is independent of p; this is the signed area (with multiplicity) enclosed by γ. Now suppose $\tilde{\gamma}$ is a Darboux transform of γ and both are closed discrete curves. Show that $A(\gamma) = A(\tilde{\gamma})$.

Exercise 2: Permutability of Darboux transforms. (5 pts)

For $z, q \in \mathbb{C} \setminus \{0\}$ let M_z^q denote the matrix

$$M_z^q = \begin{pmatrix} 1 & -z \\ -\frac{q}{z} & 1 \end{pmatrix}$$

or the associated Möbius transformation. Show that

$$M_{z_{12}-z_1}^{q_1} M_{z_1-z}^{q_2} = M_{z_{12}-z_2}^{q_2} M_{z_2-z}^{q_1}$$

if and only if

$$\mathrm{cr}(z, z_1, z_{12}, z_2) = \frac{q_2}{q_1}.$$

Exercise 3: Permutability theorem and circles (6 pts)

Consider seven points $z, z_1, z_2, z_3, z_{12}, z_{13}, z_{23}$ in $\mathbb{C}$ such that each of the three quadruples z, z_i, z_j, z_{ij} lies on a circle. For each i there is a circle through the three points z_i, z_{ij}, z_{ik} (where $\{i, j, k\} = \{1, 2, 3\}$). Show that these three circles intersect in a common point, which we might call z_{123}.

Due: **Before the tutorials** on June 29 and 30, 2016.