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Complex Analysis I

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/ComplexAnalysis/

Exercise Sheet 5

Due before the lecture on Monday, May 29, 2017.

Exercise 17: Contour integrals. (4 pts)

1. For $k \in \mathbb{Z}$, determine
$$\int_{|z|=1} \bar{z}^k dz\,.$$
2. Let γ_1 be the segment between 0 and $1+i$, and let γ_2 be the concatenation of the segment between 0 and i and the segment between i and $i+1$. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = 2(x+2iy)$. Calculate both
$$\int_{\gamma_1} f(z)dz \qquad \text{and} \qquad \int_{\gamma_2} f(z)dz\,.$$

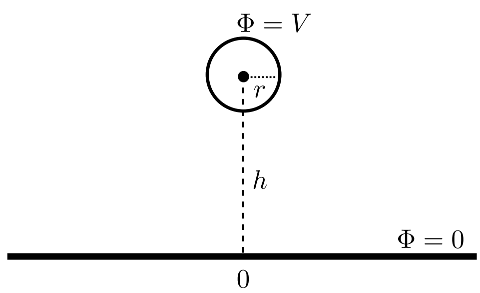
Exercise 18: Derivative of the inverse function. (4 pts)

Let $f : U \to \mathbb{C}$ be holomorphic. Show that if $f'(z_0) \neq 0$, then f has a holomorphic inverse f^{-1} locally around $w_0 := f(z_0)$. Conclude that
$$\left(f^{-1}\right)'(w_0) = \frac{1}{f'(z_0)}.$$

Exercise 19: Principal value logarithm. (4 pts)

1. Recall the principal value logarithm $\mathrm{Log}(z) = \ln(|z|) + i\mathrm{Arg}(z)$. Show that Log has derivative
$$\mathrm{Log}'(z) = \frac{1}{z}.$$
2. For $\varphi \in (-\pi, \pi)$, let R_φ denote the ray $\{re^{i\varphi} \in \mathbb{C} \mid r \in [0,\infty)\}$. Find an antiderivative for $z \mapsto \frac{1}{z}$ on $\mathbb{C} \setminus R_\varphi$.

Exercise 20: Planar electrostatics. (4 pts)



1. Let $0 < r < h$, and let C be the circle with center ih and radius r. Find a Möbius transformation that maps the real axis to the unit circle as well as the circle C to a circle $\tilde{C}$ centered at 0 of radius $\tilde{r} < 1$. Determine $\tilde{r}$.
2. Denote by A the annulus bounded by $\tilde{C}$ and the unit circle. Find the *electric potential* Ψ on A having some constant value $V \in \mathbb{R}$ on $\tilde{C}$ and 0 on the unit circle.[1]
3. On the region bounded by the circle C and the real axis, determine the electric potential Φ with boundary values V on C and 0 on the real axis.

[1] Ψ is a real valued, harmonic function.