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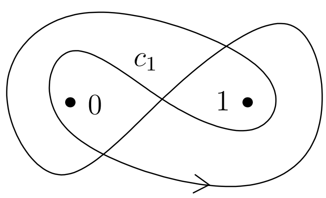
Complex Analysis I

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/ComplexAnalysis/

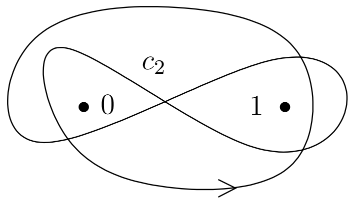
Exercise Sheet 13

Bonus Sheet – not due.

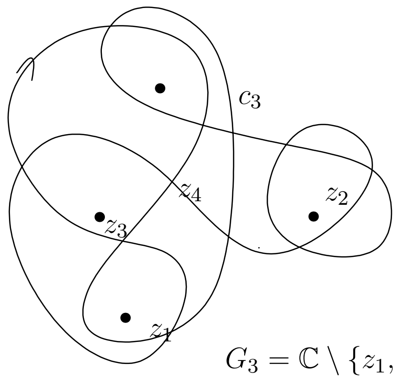
Exercise 46: Winding numbers. (4 pts)



$G_1 = \mathbb{C} \setminus \{0, 1\}$



$G_2 = \mathbb{C} \setminus \{0, 1\}$



$G_3 = \mathbb{C} \setminus \{z_1, z_2, z_3, z_4\}$

Determine the winding number of the curves c_i around each point in the complement of G_i.

Exercise 47: Residues. (4 pts)

1. Compute $\operatorname{res}(\frac{\tan z}{z^2}, 0)$.
2. Compute $\operatorname{res}(e^{z+\frac{1}{z}}, 0)$.
3. For $f(z) = \frac{\cot(\pi z)}{z-1}$, compute $\operatorname{res}(f, 0)$ and $\operatorname{res}(f, 1)$.
4. For $w \in \mathbb{C} \setminus \mathbb{Z}$ and $f(z) = \frac{\cot(\pi z)}{(z-w)^2}$ show that

$$\operatorname{res}(f, w) = -\frac{\pi}{\sin^2(\pi w)}.$$

Exercise 48: Residue theorem. (4 pts)

Verify the following integral for $0 < p < 1$:

$$\int_0^{2\pi} \frac{\mathrm{d}\theta}{1 - 2p\cos\theta + p^2} = \frac{2\pi}{1 - p^2}.$$

Hint: Show that $\int_0^{2\pi} \frac{\mathrm{d}\theta}{1-2p\cos\theta+p^2} = \int_{|z|=1} \frac{i\mathrm{d}z}{(z-p)(pz-1)}$.

Exercise 49: Integrals. (4 pts)

Let $a > 0$. Compute the following integrals using the residue theorem or related results.

1. $\displaystyle\int_0^\infty \frac{x^2}{x^4 + 6x^2 + 5}\mathrm{d}x$
2. $\displaystyle\int_{-\infty}^\infty \frac{x \sin x}{x^2 + a^2}\mathrm{d}x$