TECHNISCHE UNIVERSITÄT BERLIN
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Geometry II - SS 2017

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/GeometryII/

Exercise Sheet 02

Exercise 1: 4D consistency of discrete line congruences. (4 pts)

Prove that the 3D system governing generic discrete line congruences is 4D consistent.

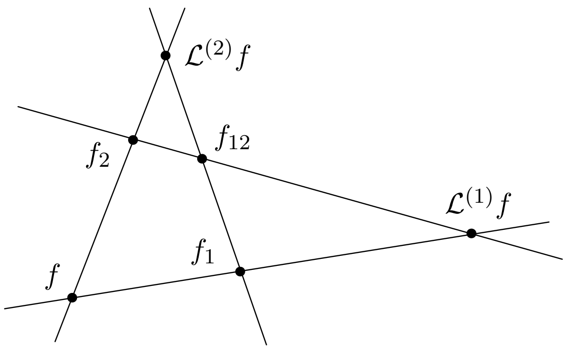
Exercise 2: Laplace transform of Q-nets. (4 pts)

Let $f : \mathbb{Z}^2 \to \mathbb{RP}^N$ be a two-dimensional Q-net. For $i = 1, 2$ the lines

$$\ell^{(i)}(u) = (f(u)f(u + e_i)) = (ff_i)$$

constitute a discrete line congruence – the so called i-th *tangent congruence* of f. Define the i-th *Laplace transform* $\mathcal{L}^{(i)}f$ of the net f as the j-th focal net of the congruence $\ell^{(i)}$, $i \neq j$, so that

$$\mathcal{L}^{(1)}f(u) = \ell^{(1)}(u) \cap \ell^{(1)}(u + e_2), \quad \mathcal{L}^{(2)}f(u) = \ell^{(2)}(u) \cap \ell^{(2)}(u + e_1).$$



Show that the Laplace transforms of f are Q-nets.

Exercise 3: 4D consistency of 2D systems. (4 pts)

Prove that 3D consistency of a discrete 2D system implies its 4D consistency.
Explain why this yields consistency in all higher dimensions.

Due Thursday, May 11, before the lecture.