TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. Alexander I. Bobenko
Jan Techter

Geometry II - SS 2017

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/GeometryII/

Exercise Sheet 03

Exercise 1: Cross-ratio and dual quadrilaterals in the complex plane. (4 pts)

Given four different point $z_1, z_2, z_3, z_4 \in \mathbb{C}$ in the complex plane, let their (complex) cross-ratio $q(z_1, z_2, z_3, z_4)$ be defined as

$$q(z_1, z_2, z_3, z_4) = \frac{(z_1 - z_2)}{(z_2 - z_3)} \frac{(z_3 - z_4)}{(z_4 - z_1)}.$$

Identify the plane $\mathbb{R}^2$ with $\mathbb{C}$ and consider two non-similar quadrilaterals in $\mathbb{C}$ whose corresponding edges are parallel.
Prove that these quadrilaterals are dual if and only if their cross-ratios are the same.

Exercise 2: Characterization of 3D Koenigs nets in terms of intersection points of diagonals. (4 pts)

Prove that a three-dimensional Q-net $f : \mathbb{Z}^3 \to \mathbb{R}^N$ is a discrete Koenigs net if and only if for every point $f = f(u)$ and for every elementary hexahedron with a vertex f, the intersection points of diagonals of the three hexahedron faces adjacent to f are collinear.

Exercise 3: Point equation of discrete Koenigs nets. (4 pts)

Show that

$$\left(\frac{1}{\nu_j} - \frac{1}{\nu_i}\right)\left(\frac{f_{ij}}{\nu_{ij}} - \frac{f}{\nu}\right) = \left(\frac{1}{\nu_{ij}} - \frac{1}{\nu}\right)\left(\frac{f_j}{\nu_j} - \frac{f_i}{\nu_i}\right)$$

and

$$\delta_i \delta_j f = \frac{\nu_j \nu_{ij} - \nu \nu_i}{\nu(\nu_i - \nu_j)} \delta_i f + \frac{\nu_i \nu_{ij} - \nu \nu_j}{\nu(\nu_j - \nu_i)} \delta_j f.$$

are equivalent.

Due Thursday, May 18, before the lecture.