TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. Alexander I. Bobenko
Jan Techter

Geometry II - SS 2017

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/GeometryII/

Exercise Sheet 06

Exercise 1: Möbius transformations in $\hat{\mathbb{C}}$. (4 pts)

Show that the orientation preserving Möbius transformations of $\mathbb{S}^2$ may be written as projective transformations of the complex projective line $\mathbb{CP}^1$. How can the form given in the homework exercise 1 of sheet 5 be deduced?

Exercise 2: Orthogonally intersecting circle. (4 pts)

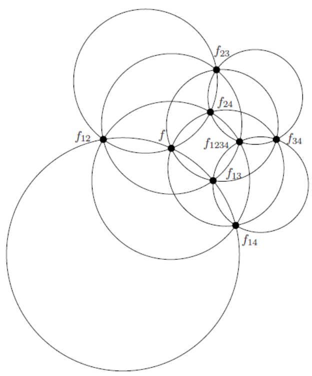
Let a sphere S and a circle C intersecting S orthogonally be given. Show that any sphere containing C is orthogonal to S.

Hint: Perform a Möbius transformation sending one of the intersection points of C with S to infinity.

Exercise 3: Clifford's chain of theorems. (4 pts)

The following three theorems constitute the beginning of an infinite sequence (Clifford's chain of theorems).

CLIFFORD'S FIRST THEOREM. Let C_1, C_2, C_3, C_4 be four circles in general position in a plane, with a common point f. Let f_{ij} be the second intersection point of the circles C_i and C_j. Let C_{ijk} denote the circle through f_{ij} , f_{ik}, f_{jk}. Then the four circles C_{123}, C_{124}, C_{134}, C_{234} all pass through one point f_{1234}; see figure below.



CLIFFORD'S SECOND THEOREM. Let $C_1, \ldots, C_5$ be five circles in general position in a plane, with a common point f. Then the five points f_{1234}, f_{1235}, f_{1245}, f_{1345}, f_{2345} all lie on one circle C_{12345}.

CLIFFORD'S THIRD THEOREM. Let $C_1, \ldots, C_6$ be six circles in general position in a plane, with a common point f. Then the six circles C_{12345}, C_{12346}, C_{12356}, C_{12456}, C_{13456}, C_{23456} all pass through one point f_{123456}.

Hint: Prove these theorems, by restricting Cox's chain of theorems (see exercise 3 on sheet 4) to a sphere.

Due Thursday, June 08, before the lecture.