TECHNISCHE UNIVERSITÄT BERLIN
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Geometry II - SS 2017

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/GeometryII/

Exercise Sheet 08

Exercise 1: Cross-ratios of four adjacent quadrilaterals. (4 pts)

Let $f : \mathbb{Z}^2 \to \mathbb{R}^N$ be a two-dimensional circular net. Show that, if the cross-ratios $q = q(f, f_1, f_{12}, f_2)$ of its elementary quadrilaterals satisfy

$$q \cdot q_{-1,-2} = q_{-1} \cdot q_{-2},$$

then f is discrete isothermic.

Exercise 2: Factorization of the cross-ratios and metric coefficient. (4 pts)

Let $f : \mathbb{Z}^m \to \mathbb{R}^N$ be a discrete isothermic net with metric coefficient $s : \mathbb{Z}^m \to \mathbb{R}^*$. Define a labelling α of edges of $\mathbb{Z}^m$ such that $\|f_i - f\|^2 = \alpha_i s s_i$, $(i = 1, \dots, m)$. Then the cross-ratios of its elementary quadrilaterals are factorized as $q(f, f_i, f_{ij}, f_j) = \frac{\alpha_i}{\alpha_j}$.

Hint: Similarity of triangles as in the proof of the theorem on metric coefficients.

Exercise 3: Discrete Darboux transformation. (4 pts)

Adapt the general definition of a discrete fundamental transformation for discrete isothermic nets:

Definition. (Discrete Darboux transformation)

A pair of discrete isothermic nets $f, f^+ : \mathbb{Z}^m \to \mathbb{R}^N$ with likewise factorized cross-ratios, such that $q(f, f_i, f_{ij}, f_j) = \frac{\alpha_i}{\alpha_j}$ and $q(f^+, f_i^+, f_{ij}^+, f_j^+) = \frac{\alpha_i}{\alpha_j}$ holds, is related by a *Darboux transformation* if . . .

The net f^+ is called a *Darboux transform* of the net f with the parameter . . . .

You may also think about initial data which determines a Darboux transform f^+ of a given discrete isothermic net f uniquely and about permutability.

Hint: Theorem on consistency of discrete isothermic nets.

Due Thursday, June 29, before the lecture.