

Exercise Sheet 09

Exercise 1: Oriented area of a k -gon. (5 pts)

Consider directions $V = \{[v_1], \dots, [v_k]\}$ in \mathbb{RP}^1 and the corresponding $(k-2)$ -dimensional vector space $\tilde{\mathcal{P}}(V)$ of k -gons (p_1, \dots, p_k) , $p_i \in \mathbb{R}^2$ with parallel edges factored by translation. Show that the oriented area

$$A(P) = \frac{1}{2} \sum_{i=1}^k \det(p_i, p_{i+1}),$$

is a quadratic form on $\tilde{\mathcal{P}}(V)$.

Exercise 2: Area form of quadrilaterals. (2 pts)

Show that if a crossed quadrilateral $ABCD$ has signed area 0, then the diagonals AC and BD are parallel.

Exercise 3: Dual quadrilaterals of different type. (5 pts)

Consider following three types of quadrilaterals:

- (a) embedded convex
- (b) embedded non-convex
- (c) non-embedded (crossed)

For which combinations of types is it possible to have quadrilaterals P_1, P_2 with parallel corresponding edges, such that the mixed area satisfies $A(P_1, P_2) = 0$?