TECHNISCHE UNIVERSITÄT BERLIN
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Geometry II - SS 2017

http://www3.math.tu-berlin.de/geometrie/Lehre/SS17/GeometryII/

Exercise Sheet 10

Exercise 1: Weirestrass representation for circular minimal surfaces. (8 pts)

Derive the following discrete Weierstrass representation for circular minimal surfaces:

$$\delta_1 f = \alpha_1 \Re\left(\frac{1-g_1 g}{g_1-g}, \frac{i(1+g_1 g)}{g_1-g}, \frac{g_1+g}{g_1-g}\right),$$
$$\delta_2 f = \alpha_2 \Re\left(\frac{1-g_2 g}{g_2-g}, \frac{i(1+g_2 g)}{g_2-g}, \frac{g_2+g}{g_2-g}\right),$$

where $g : \mathbb{Z}^2 \to \mathbb{C}$ is a discrete holomorphic map, i.e., a solution of the cross-ratio equation

$$q(g, g_1, g_{12}, g_2) = \frac{\alpha_1}{\alpha_2};$$

Hint: The isothermic Gauss map $n : \mathbb{Z}^2 \to \mathbb{S}^2$ is given by the stereographic projection of g,

$$(n_1 + in_2, n_3) = \left(\frac{2g}{|g|^2+1}, \frac{|g|^2-1}{|g|^2+1}\right).$$

Exercise 2: Discrete Enneper surface. (4 pts)

Derive an explicit formula for the discrete Enneper minimal surface via the discrete Weierstrass representation by applying it to the standard square grid (which is obviously discrete isothermic).

Due Thursday, July 13, before the lecture.