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Complex Analysis I

http://www3.math.tu-berlin.de/geometrie/Lehre/SS18/ComplexAnalysis/

Exercise Sheet 2

Due before the tutorials on Monday, May 7, 2018.

Exercise 1: Contour integrals. (4 pts)

1. For $k \in \mathbb{Z}$, determine
$$\int_{|z|=1} \overline{z}^k dz\,.$$
2. Let $\gamma_1 = [0, i+1]$ and $\gamma_2 = [0, i] + [i, i+1]$. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = 2(x + 2iy)$. Calculate both
$$\int_{\gamma_1} f(z)dz \qquad \text{and} \qquad \int_{\gamma_2} f(z)dz\,.$$

Exercise 2: Line integrals. (4 pts)

Let $U \subseteq \mathbb{C}$ be a domain, and let $f : U \to \mathbb{C}$ be holomorphic. Show that for any closed curve γ in U, the integral
$$\int_\gamma \overline{f(z)} f'(z)dz$$
is purely imaginary.

Exercise 3: Integral estimate. (4 pts)

Let $g : [a, b] \to \mathbb{C}$ and $f : U \to \mathbb{C}$ be continuous functions, $U \subset \mathbb{C}$, and $\gamma : [c, d] \to \mathbb{C}$ be a curve. Let $\max_\gamma |f| := \max\{|f(\gamma(t))| \mid t \in [c, d]\}$. Prove that
$$\left|\int_a^b g(t)dt\right| \leq \int_a^b |g(t)|dt \quad \text{and} \quad \left|\int_\gamma f(z)dz\right| \leq L(\gamma) \max_\gamma |f|,$$
where $L(\gamma)$ denotes the length of γ.

Exercise 4: Complex conjugation. (4 pts)

Let $D \subseteq \mathbb{C}$ be a domain, $f : D \to \mathbb{C}$ a continuous map. Show that:

1. $\overline{\int_\gamma f(z)dz} = \int_{\overline{\gamma}} \overline{f(\overline{z})}dz$ for every curve γ in D.
2. $\overline{\int_{|z|=1} f(z)dz} = -\int_{|z|=1} \overline{f(z)}\frac{dz}{z^2}$ if $\{z \in \mathbb{C} \mid |z| = 1\} \subseteq D$.