

## Exercise Sheet 9

### Exercise 1: Discrete Gauss curvature.

(5 pts)

- (a) Show that a convex polyhedron has positive Gauss curvature at every vertex.
- (b) Give an example of a non-convex polyhedron with positive Gauss curvature at every vertex to show that the converse is not true.

### Exercise 2: Simplicial faces and simple vertices of a convex polyhedron. (5 pts)

The degree  $\deg(v)$  of a vertex  $v$  of a discrete surface is the number of edges adjacent to it. The degree  $\deg(f)$  of a face  $f$  of a discrete surface is the number of edges its boundary consists of.

Show that every convex polyhedron has a triangle face or a vertex of degree 3 (or both). Even stronger: Show that the number of triangle faces plus the number of vertices of degree 3 is at least eight, so there are at least four triangle faces or four vertices of degree 3.

**Hint:** Use the double counting formula:

$$\sum_{v \in V} \deg(v) = \sum_{f \in F} \deg(f) = 2|E|$$

and the fact that a convex polygon is topologically a sphere.

### Exercise 3: Straight curves on a polyhedral surface. (6 pts)

Let  $S$  be a polyhedral surface with vertices  $V$ . For each point  $P \in S \setminus V$ , there is a neighbourhood which is isometric to an open disc in  $\mathbb{R}^2$ . (For points in the interior of a 2-face this is trivial, for points in the interior of an edge one just has to "unfold" along the edge). Let  $\gamma$  be a curve in  $S \setminus V$ . The curve  $\gamma$  is called *straight*, if all images of  $\gamma$  under the above described isometries are straight line segments.

- (a) Express in terms of angles how a straight curve  $\gamma$  crosses the edges of  $S$ .
- (b) Find a polyhedral surface  $S$  which contains two points  $P$  and  $Q$ , such that there are infinitely many straight lines between  $P$  and  $Q$  (with different traces).