

Exercise Sheet 1

Exercise 1. Confocal conics in a Euclidean plane. (4 pts)

Given a hyperbola and an ellipse with same foci F_1, F_2 , intersecting in a point X . Show that the two curves intersect orthogonally in X .

Exercise 2. Dandelin spheres. (4 pts)

In the lecture it was illustrated how to prove that the intersection of a cone with certain planes are ellipses using so-called Dandelin spheres (spheres which are tangent to the cone and the plane, touching in the foci of the ellipse). Can you adapt this proof for planes that intersect a cone in either two components or a curve that is not closed in \mathbb{R}^3 , showing that the intersection is a hyperbola and parabola, resp.

Exercise 3. Locus of midpoints of touching circles. (4 pts)

Prove the following theorem from the lecture:

The locus of midpoints of all circles touching a fixed circle C around F_1 and passing through a point F_2 in the interior of C is an ellipse with foci F_1, F_2 .

What if F_2 is chosen outside the circle? What if you replace C by a line? Prove your claim.