

Exercise Sheet 11 (the last one)

Exercise 1: Contact elements. (3 pts)

Show that for each contact element l in the Lie quadric and a Lie sphere $s \notin l$, there is a unique sphere $r \in l$ such that s and r are in oriented contact.

Exercise 2: Cocyclic points. (3 pts)

Show that four points in $\widehat{\mathbb{R}^N}$ lie on a circle or line if and only if the corresponding points in the intersection of the Lie quadric \mathcal{L} and $P(e_{n+3}^\perp) \subset \mathbb{R}P^{N+1,2}$ are coplanar.

Exercise 3: Lie transformation of contact elements. (6 pts)

Let $f : \mathbb{R}P^4 \rightarrow \mathbb{R}P^4$ be a projective transformation s.t. $f([p]) = [Mp]$ with

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cosh w & 0 & 0 & \sinh w \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \sinh w & 0 & 0 & \cosh w \end{pmatrix}$$

w.r.t. the basis $B = \{e_1, \dots, e_n, e_0, e_\infty, e_{n+3}\}$.

1. Show that t is a Lie transformation.
2. What is the image of an oriented circle with center $c \in \mathbb{R}^2$ and radius $r \in \mathbb{R}$ under f ?
3. Let

$$\gamma, \eta : \mathbb{R} \rightarrow \mathbb{R}^2, \quad \gamma(t) = (t, 0), \eta(t) = (0, t)$$

be two parametrized curves in the plane. Identifying each curve with a one-parameter family of contact elements in the Lie quadric, what is the image of γ resp. η under f ? Give the resp. one-parameter family of point sphere representatives in the image of the contact elements.