TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Boris Springborn
Isabella Retter

Geometry I
Summer 2019

https://www3.math.tu-berlin.de/geometrie/Lehre/SS19/GeometryII/

Exercise Sheet 2

Exercise 1. (+ + - -) quadric. (4 pts)

Let B be the symmetric bilinear form on $\mathbb{R}^4$ representing the surface $x^2 + y^2 = z^2 + w^2$ – a quadric of signature $(++--)$.

- Sketch the quadric surface $P(B(u,u) = 0) \subset \mathbb{R}P^3$.
- Show that the polar plane of any point P, not lying on the quadric, cuts the quadric in a non-degenerate conic.
- Find and sketch the polar plane of the point $S = (0, 0, 1, 0)$.
- Show that $T = (1, 0, 0, 1)$ lies on the surface, and find the equations of the two lines lying on the surface which pass through T.

Exercise 2. Polarity of lines in $\mathbb{R}P^3$. (4 pts)

Let $\mathcal{Q}$ be a non-degenerate quadric in $\mathbb{R}P^3$, $l \subset \mathbb{R}P^3$ a line and $l^\perp \subset \mathbb{R}P^3$ its polar. Show that all polar planes of points on l pass through $l^\perp$.

Exercise 3. Degenerate quadrics. (4 pts)

Let q be a degenerate, but non-vanishing quadratic form on $\mathbb{R}^{n+1}$, which defines the quadric $\mathcal{Q} \subset \mathbb{R}P^n$. Let b be the corresponding symmetric bilinear form and denote $U_0 = \ker q = \{u \in \mathbb{R}^{n+1} | b(u,v) = 0 \; \forall \, v \in \mathbb{R}^{n+1}\}$. Consider any complementary subspace U_1 of U_0, such that $\mathbb{R}^{n+1} = U_0 \oplus U_1$.
Denote $\mathcal{Q}_1 \subset P(U_1)$ the non-degenerate quadric $\mathcal{Q}_1$ that is defined by the restriction $q|_{U_1}$. Under the assumption $\mathcal{Q}_1 \neq \emptyset$, show that $\mathcal{Q}$ is the union of all lines joining any point in $P(U_0)$ with any point on $\mathcal{Q}_1$. Draw a sketch that illustrates this decomposition in $\mathbb{R}P^3$. What happens if $\mathcal{Q}_1$ is empty?

Due: Tuesday, 30.04.2019 before the lecture