TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry I
Summer 2019

https://www3.math.tu-berlin.de/geometrie/Lehre/SS19/GeometryII/

Exercise Sheet 3

Please note that the Wednesday morning tutorial has moved to Tuesday morning, see course website for details. Starting April 30th, tutorials will be held Tuesday 8:30-10 and Wednesday 14:14-15:45. As May 1st is a holiday, everyone is welcome to join the tutorial on Tuesday, 30.4., 8:30 am.

For exercises 1 and 2 (see also next page), we need the definition of *inversion in a circle*:

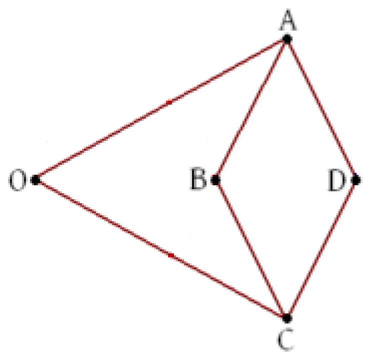
In the Euclidean plane, inversion in a circle with center O and radius R is given by the map

$$A \mapsto A' = O + \frac{R^2}{||A-O||^2}(A-O).$$

Note that A and A' lie on the same ray emanating from O and $||A-O||\,||A'-O|| = R^2$.

Exercise 1. (4 pts)

There is a simple mechanism, called the *Peaucellier-Lipkin linkage* which mechanically performs inversion on a circle. It consists of six rods: two of length a joining a fixed point O to two opposite corners A and C of a rhombus $ABCD$ with side length $b < a$, with hinges at all four corners. Show: As B moves along some curve, D moves along the curve obtained by inversion on a circle (and vice versa).



(see reverse)

Exercise 2. (4 pts)

Let C be a circle in the Euclidean plane with center O and radius r. The *power of a point P with respect to the circle C* is defined as

$$p_C(P) = d^2 - r^2,$$

where d is the distance from the center O to P. Show: If a line through P intersects the circle in the points A and B, then the power of P with respect to C is the product of the lengths PA and PB, taken negatively if P is between A and B.

Exercise 3. (4 pts)

Let q be a quadratic form on $\mathbb{R}P^n$ with signature $(n, 1)$ with $n \geq 2$, and let

$$\mathcal{Q} = \{[x] | q(x) = 0\}$$

be the quadric defined by q. Show that the following statements are equivalent:

1. Every line through $A = [a]$ intersects $\mathcal{Q}$ in two points.
2. $q(a, a) < 0$.

Due: Tuesday, 07.05.2019 before the lecture