TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Boris Springborn
Isabella Retter

Geometry I
Summer 2019

https://www3.math.tu-berlin.de/geometrie/Lehre/SS19/GeometryII/

Exercise Sheet 5

Exercise 1. Orthogonal spheres. (4 pts)

Show that there are $n+1$ but no $n+2$ pairwise orthogonal hyperspheres in $\widehat{\mathbb{R}^n}$.

Exercise 2: Orthogonal group. (4 pts)

Let $\langle \cdot, \cdot \rangle$ denote the indefinite scalar product of $\mathbb{R}^{p,q}$, and for a non-isotropic vector $w \in \mathbb{R}^{p,q}$ let the map $R_w : \mathbb{R}^{p,q} \to \mathbb{R}^{p,q}$ be defined by

$$R_w(x) = x - 2\frac{\langle x, w \rangle}{\langle w, w \rangle} w.$$

Show:
1. $R_w \in O(p,q)$.
2. If $A \in O(p,q)$, $v \in \mathbb{R}^{p,q}$ and $w = Av - v$ is non-isotropic, then $R_w(v) = Av$ and $R_w(Av) = v$.

Exercise 3: Skew symmetric bilinear forms. (4 pts)

Let ω be a skew-symmetric non-degenerate bilinear form on an n-dimensional real vector space V. This means

- $\omega(v,w) = -\omega(w,v)$ for all $v, w \in V$,
- for each $v \in V \setminus \{0\}$ there is a $w \in v$ with $\omega(v,w) \neq 0$.

Show that there is a basis $(v_1, \ldots, v_n)$ of V such that the matrix $\Omega = (\omega(v_i, v_j)_{ij}) \in \mathbb{R}^{n \times n}$ is

$$\Omega = \begin{pmatrix} 0 & 1 & & & & & \\ -1 & 0 & & & & & \\ & & 0 & 1 & & & \\ & & -1 & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & 1 \\ & & & & & -1 & 0 \end{pmatrix}.$$

In particular, show that n is even.

Due: Tuesday, 21.05.2019 before the lecture