TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry I
Summer 2019

https://www3.math.tu-berlin.de/geometrie/Lehre/SS19/GeometryII/

Exercise Sheet 6

Exercise 1. Fixpoints of orientation-reversing Möbius transformations. (4 pts)

Show that an orientation-reversing Möbius transformation of $\widehat{\mathbb{C}}$ that is not the identity has either $0, 1$ or 2 fixed points or a whole circle of fixed points. Give an example for each case.

Exercise 2: Circle inversion. (4 pts)

Show that any pair of circles in $\widehat{\mathbb{C}}$ can be mapped by a Möbius transformation either to a pair of straight lines, or to a pair of concentric circles.

Exercise 3. Steiner porism. (6 pts)

A *Steiner chain* is a finite sequence of circles $C_1, ..., C_n$, where each circle is tangent to two given non-intersecting circles $A,\ B$, and every circle in the chain is tangent to the previous and next circle in the chain. If the first and n-th circle are also tangent, the chain is called closed.
Prove the following theorem, called *Steiner Porism*:

If there exists at least one closed Steiner chain of n circles for two given non-intersecting circles A and B, then there is an infinite number of closed Steiner chains of n circles; and any circle tangent to A and B in the same way is a member of such a chain.

Hint: Exercise 2.

Due: Tuesday, 28.05.2019 before the lecture