

Exercise Sheet 9

Exercise 1: Reverse contact. (4 pts)

We say that two oriented hyperspheres are in reverse contact if they are in oriented contact after a change of orientation of one of them. Show that a Lie transformation that maps every pair of oriented hyperspheres in reverse contact to oriented hyperspheres in reverse contact maps point spheres to point spheres (and hence comes from a Möbius transformation).

Exercise 2: Conic section. (3 pts)

Let C_1, C_2 be two oriented circles in \mathbb{R}^2 that are in oriented contact. Show that the locus of midpoints of oriented circles, oriented lines and points that are in oriented contact with C_1 and in reverse contact with C_2 is a conic section.

Exercise 3. Lines in the Lie quadric and oriented contact. (5 pts)

1. Let $[p] \in \mathbb{R}P^{n+2}$ with $q_{Lie}(p) < 0$ and let l be a line in the Lie quadric \mathcal{L} . Show that l intersects p^\perp at exactly one point.
2. Show that every point sphere is obtained as the intersection of \mathcal{L} with e_{n+3}^\perp and oriented hyperplanes correspond to points in the intersection of \mathcal{L} with e_0^\perp .
3. Show that every line l contained in the Lie quadric contains exactly point sphere. Furthermore, if the point sphere is a proper point (i.e., is not the point at ∞), then l contains exactly one oriented hyperplane.