1 Correction to Aufgabe 4-i) from Blatt 1, Geometrie I

Desargues configurations The *Desargues configuration* refers to the configuration of the Theorem of Desargues: the two triangles ABC and $A'B'C'$ are such that the joining lines of corresponding vertices AA', BB' and CC' meet in a point P (the so-called *center*); consequently, the intersections of corresponding pairs of sides $C'' = (AB, A'B')$, $A'' = (BC, B'C')$, and $B'' = (AC, A'C')$ lie in a line, the so-called *axis*. When the joins of corresponding vertices meet in a point, we say the triangles are *perspective in a point*; when the intersections of corresponding sides lie in a line, we say the triangles are *perspective in a line*. In class we proved the version of Desargues that claims: *If two triangles are perspective in a point, they are also perspective in a line.*

i) **Corrected exercise!** The converse of Desargues Theorem is: *If two triangles are perspective in a line, they are perspective in a point.* Prove the converse by carrying out the following steps:

   a) Let 3 lines in $P^2(\mathbf{R})$be given by $L_i : a_i x + b_i y + c_i z = 0$, for $i \in \{0, 1, 2\}$. Show that the three lines are concurrent (have a common point) if and only if the 3 vectors $l_i := (a_i, b_i, c_i)$ are linear dependent in $\mathbb{R}^3$. [Remember: the common point must be a vector $(x, y, z) \neq (0, 0, 0)$.]

   b) When the three lines are concurrent, show that you can assume that the linear dependence among the three vectors l_i takes the form $l_0 + l_1 = l_2$. [Hint: $ax + by + cz = 0$ and $\lambda ax + \lambda by + \lambda cz = 0$ for $\lambda \neq 0$ describe the same line.]

   c) Finally, using b), show how to turn around the proof of Desargues given in class, in order to prove its converse.

   d) Show that the medians of a triangle are concurrent.