1 Remarks on notation for Projective Geometry

- To avoid unwanted connotations of the word *infinity*, the extra points, lines, etc, added to obtain projective geometry will be referred to in the assignment sheets as *ideal* points, lines, etc.
- Points and lines
  1. Points will be labeled with uppercase letters. Lines will be labeled with lowercase letters. Planes will be labeled with Greek letters.
  2. The line which joins two points A and B will be denoted AB. The point of intersection of two lines l and m will be denoted lm. Similar rules apply for planes.
  3. $(AB)(CD)$ denotes the point of intersection of the line joining A and $B$with the line joining C and D, etc
- **C**oordinates
  1. A vector in $\mathbb{R}^{n+1}$ is represented by its coordinates as $(x_0, x_1, ...x_n)$.
  2. We identify the one-dimensional subspace spanned by this vector as the point $P \in P^n(\mathbf{R})$and write $P = [(x_0, x_1, ...x_n)]$.
  3. As a matter of convenience, we usually omit the square brackets in the above notation for $P^n(\mathbf{R})$and use the same notation for $P^n(\mathbf{R})$as for $\mathbb{R}^{n+1}$, depending on the context to make clear which is meant. In the context of $P^n(\mathbf{R})$, you have to remember that the coordinates can be multiplied by any non-zero factor without changing its value.
  4. We need homogeneous coordinates to work with all of $P^n(\mathbf{R})$. Sometimes however we are working with $\mathbb{R}^n \subset P^n(\mathbf{R})$.
  5. **Example** Consider O in $P^2(\mathbf{R})$defined by $O = (x, y, z) \mid z \neq 0$. For $(x, y, z) \in O$, choose representative $(\frac{x}{z}, \frac{y}{z}, 1)$. Then O is a copy of $\mathbb{R}^2$ and we can refer to the coordinates $(\frac{x}{z}, \frac{y}{z}, 1)$ or $(\frac{x}{z}, \frac{y}{z})$ as the *non-homogeneous* coordinates for this copy of $\mathbb{R}^2$.
  6. Unless otherwise stated, in the assignment sheets for this course the **last** coordinate will be the one used to extract non-homogeneous coordinates. [The lecture notes by Hitchin use the first coordinate for this purpose].
  7. Another way to think about homogeneous vs non-homogeous coordinates, is that I can choose one coordinate as a special coordinate, say x_j. Then if $x_j = 0$, I leave the coordinates alone; otherwise I multiply by $\frac{1}{x_j}$. The resulting coordinates have either $x_j = 0$ of $x_j = 1$.