1 Matrix form for the operators k and $\hat{k}$

First point: the basis of $\wedge^3(V)$ dual to the standard basis of V is

$$\begin{aligned} E_1 &:= e_2 \wedge e_3 \wedge e_4 \\ E_2 &:= -e_1 \wedge e_3 \wedge e_4 \\ E_3 &:= e^1 \wedge e_2 \wedge e_4 \\ E_4 &:= -e_1 \wedge e_2 \wedge e_3 \end{aligned}$$

Because: $e_i \wedge E_i(det) = 1$, there must be minus signs in the second and fourth elements!
Then, one calculates that the matrix form for $k : V \to \wedge^3(V) \cong V^*$ with $k(v) := a \wedge v$ is:

$$M := \begin{pmatrix} 0 & a_{34} & a_{24} & a_{23} \\ -a_{34} & 0 & a_{14} & -a_{13} \\ -a_{24} & -a_{23} & 0 & a_{12} \\ -a_{23} & a_{13} & -a_{12} & 0 \end{pmatrix}$$

Now, define a second matrix

$$\hat{M} := \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} \\ -a_{12} & 0 & a_{23} & -a_{24} \\ -a_{13} & -a_{23} & 0 & a_{34} \\ -a_{14} & a_{24} & -a_{34} & 0 \end{pmatrix}$$

Then one verifies that $M\ \hat{M} = -\omega(a,a)Id$, so projectively they are inverses of each other. The matrices M and $\hat{M}$ can be thought of as polarizing operators. M takes a point and gives the polar plane, while $\hat{M}$ takes a plane and gives the polar point. Being inverses means that the polarity is a symmetric relation: the polar of the polar is the original element.
Because of the skew symmetry $a_{ij} = -a_{ji}$, one can create many different forms of the above matrices. The above forms are characterized by the restriction that $i < j$ for all the a_{ij} which are used.
Example: given the first coordinate plane $[1,0,0,0]$, its polar point is
$M(1,0,0,0)^t = (0, -a_{12}, -a_{13}, -a_{14})$.
You can connect this to what we already did in the case that $a \in Q$, that is, a represents a line. Then the polar of a point is simply the plane spanned by a and the point, while the polar of a plane is simply the point of intersection of the line with the plane. And, the polar operation is not well defined for points that lie on the line and planes that pass through the line.
Example: Let g be the line joining $u = (1,0,0,0)$ and $v = (0,1,0,0)$. Then the Plücker coordinates of g are $(1,0,0,0,0,0)$ and one verifies easily that if $w = u + tv$ then $Mw^t = (0,0,0,0)^t$, which doesn't represent a plane at all! That is, the polar of such a point is not well-defined.
In all applications of these matrices, it is usually clear that $a \notin Q$ so that it really is a 1:1 correlation of points and planes.