Subdivision in periodic spaces

Kristoffer Josefsson (josefsso@math.tu-berlin.de)

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Introduction

The task is to implement common approximating subdivision schemes (Loop, $\sqrt{3}$, Catmull-Clark etc.) in periodic spaces in Java using jTem and jReality.

Given a discrete group $\Gamma \subset E(3)$ of the Euclidean group of isometries of $\mathbf{E}^3$, we have that $M = \mathbf{E}^3/\Gamma$ is a manifold iff Γ is fixed-point free (i.e. torsion free).

Code for generating these groups are provided by the `discretegroup` package in jTem.

The implementation should use the `SymmetricHEDS` class in the `halfedgetools` package of jTem. This class represents a periodic mesh by a half-edge data structure (V, E, F), an embedding $f\colon V \to \mathbf{R}^3$ into the fundamental domain of M together with a set of 1-cycles C and a map $g\colon C \to \Gamma$ attaching for each 1-cycle a group element of Γ.

The condition that Γ is torsion free ensures that for each vertex it is possible to layout a neighborhood into the flat $\mathbf{R}^3$ to perform the subdivision.

Tasks

- Implement the subdivision schemes in a modular way, separating topological changes and embedding changes. This means implementing the proper adapter setup so that the the subdivision code also works for meshes without symmetry (i.e. the standard `StandardHEDS`).
- Exemplify with at least two surfaces in different spaces:
  - SchwartzP surface in $\mathbf{T}^3$
  - Other surface in space involving reflections etc.
- Visualize the result using the existing jReality framework.
- Write proper documentation.

Possible generalizations

- If time permits, it would be nice to implement also the classical *interpolating* subdivision schemes.
- Generalize the setup to ambient space being S^3 and H^3.