TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. Alexander Bobenko
Felix Günther

Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

Exercise Sheet 1

Exercise 1. (7 points)

Let $(\mathcal{R}, g)$ be an orientable two-dimensional Riemannian manifold. Show that for conformal coordinates $w, z : U \subset \mathcal{R} \to \mathbb{C}$ (i.e., $g = e^{\phi} dz d\overline{z} = e^{\widetilde{\phi}} dw d\overline{w}$) the transition function $w \circ z^{-1}$ is holomorphic or antiholomorphic, and that $w \circ z^{-1}$ is holomorphic if and only if w and z determine the same orientation on $\mathcal{R}$.

Exercise 2. (7 points)

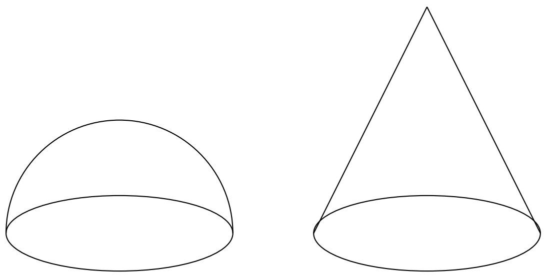
Let $f : U \subset \mathbb{C} \to \mathbb{C}$ be a local diffeomorphism. Show that f is holomorphic if and only if f preserves the orientation and the angles.

Exercise 3. (6 points)

Equip the following two closed surfaces with a complex structure:

(a) the surface consisting of a hemisphere and a disk glued along their common boundary,

(b) the surface consisting of a right circular cone and a disk glued along their common boundary.



(a) (b)

Due: Tutorial on 31 October, 2012.