TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. Alexander Bobenko
Felix Günther

Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

`www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/`

Exercise Sheet 10

Exercise 1. (7 points)

Let B be a complex symmetric $g \times g$-matrix with $\mathrm{Re}(B)$ being negative definite. Show that the theta function

$$\theta(z) := \sum_{m \in \mathbb{Z}^g} \exp\left(\frac{1}{2}(Bm, m) + (z, m)\right)$$

converges absolutely and uniformly on any compact subset of $\mathbb{C}^g$.

Exercise 2. (7 points)

a) Let $\mathcal{R}$ be a compact Riemann surface of genus g, $P \in \mathcal{R}$, and D_g the set of positive divisors of degree g. Prove that the differential of the Abel map $\mathcal{A}_P : D_g \to Jac(\mathcal{R})$ has non-trivial kernel at the divisor $D_R \in D_g$ if and only if D_R is special.

b) When is the solution to the Jacobi inversion problem unique?

Exercise 3. (6 points)

Construct a non-constant meromorphic function on a compact Riemann surface $\mathcal{R}$ of genus g in terms of Riemann theta functions $\theta(\mathcal{A}_{P_0}(\cdot) - d)$ choosing appropriate vectors $d \in \mathbb{C}^g$.

Hint: Consider quotients of theta functions with different d's.

Due: Tutorial on 23 January, 2013.