Technische Universität Berlin
Institut für Mathematik

Prof. Dr. Alexander Bobenko
Felix Günther

Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

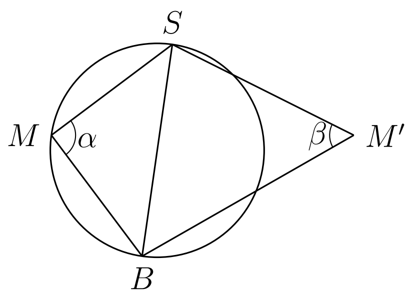
Exercise Sheet 11

Exercise 1. (7 points)

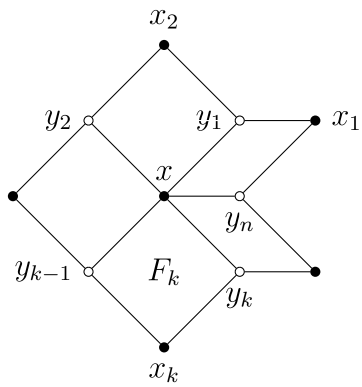
Let $D = \Gamma \cup \Gamma^*$ be a simply-connected strongly regular bipartite quad-graph together with a discrete complex structure defined by real weights ν on the edges of Γ and Γ^*. Given a discrete harmonic function $h : V(\Gamma) \to \mathbb{R}$, construct a discrete holomorphic function $f : V(D) \to \mathbb{C}$ such that $f(x) = h(x)$ for all $x \in V(\Gamma)$. How unique is f?

Exercise 2. (6 points)

Let $\triangle BMS$ and $\triangle BM'S$ be two non-overlapping triangles in the plane sharing the edge BS. Denote by α and β the non-oriented angles $\angle BMS$ and $\angle BM'S$, respectively. Prove that M' lies outside the circumcircle of $\triangle BMS$ iff $\cot(\alpha) + \cot(\beta) > 0$.



(a) Exercise 2: Triangles $\triangle BMS$ and $\triangle BM'S$



(b) Exercise 3: Star of vertex x

Exercise 3. (7 points)

Let $D = \Gamma \cup \Gamma^*$ be a strongly regular bipartite quad-graph together with a discrete complex structure defined by complex weights ν on the edges of Γ and Γ^*.
For $H : V(D) \to \mathbb{R}$, we define the *discrete Laplacian* $\triangle H$ by $2\triangle H(x)$ being

$$\sum_{F_k \ni v} \frac{1}{\mathrm{Re}\,(\nu(x,x_k))} \left(|\nu(x,x_k)|^2 \left(H(x_k) - H(x)\right) + \mathrm{Im}\,(\nu(x,x_k)) \left(H(y_k) - H(y_{k-1})\right) \right),$$

where we sum over all faces F_k having v as a vertex and use the notation of Figure (b).
Show that real and imaginary part of a discrete holomorphic function $f : V(D) \to \mathbb{C}$ are discrete harmonic, i.e., $\triangle \mathrm{Re}(f)(v) = \triangle \mathrm{Im}(f)(v) = 0$ for all inner vertices $v \in V(D)$.

Due: Tutorial on 30 January, 2013.