TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

Exercise Sheet 2

Exercise 1. (4 points)

Let $Z \subset \mathbb{R}^3$ be the infinite cylinder of radius 1, furnished with the complex structure defined by the Riemannian metric on Z which is induced by the Euclidean metric in $\mathbb{R}^3$. Show that Z is a Riemann surface with punctures.

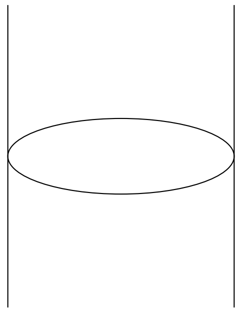

Figure 1: Cylinder Z

Exercise 2. (4 points)

Let $\mathcal{R}$ be a Riemann surface and $f : \mathcal{R} \to \hat{\mathbb{C}}$ be a non-constant meromorphic function on $\mathcal{R}$. Describe the complex structure (local coordinates) on $\mathcal{R}$ in terms of f.

Exercise 3. (6 points)

Show that any compact Riemann surface of an elliptic curve of degree 3 is biholomorphic to a compact Riemann surface of an elliptic curve of degree 4.

Exercise 4. (6 points)

Let

$$f(x, y, z) = \sum_{i,j,k \geq 0} c_{ijk} x^i y^j z^k$$

be a homogeneous polynomial of degree $n = i + j + k$. We call the *plane projective curve*

$$\mathcal{C} := \left\{ [x : y : z] \in \mathbb{C}\mathrm{P}^2 | f(x, y, z) = 0 \right\}$$

non-singular if $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})_P \neq (0, 0, 0)$ for all points $P \in \mathcal{C}$.
Show that a non-degenerate conic $\mathcal{C}$ ($n = 2$) defines a non-singular plane projective curve, and give a complex structure on $\mathcal{C}$.
For a non-degenerate conic $\mathcal{C}$ find a biholomorphic function $g : \mathcal{C} \to \mathbb{C}\mathrm{P}^1 \cong \hat{\mathbb{C}}$.
Hint: Project $\mathcal{C}$ from a point of $\mathcal{C}$ to a $\mathbb{C}\mathrm{P}^1 \subset \mathbb{C}\mathrm{P}^2$.

Due: Tutorial on 7 November, 2012.