TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

Exercise Sheet 3

Exercise 1. (7 points)

An *elliptic function* is a non-constant doubly periodic meromorphic function $f : \mathbb{C} \to \hat{\mathbb{C}}$. Poles are counted with multiplicities in a fundamental parallelogram.

(a) Show that there exists no elliptic function with at most one pole.

(b) Let f be an elliptic function with exactly one pole which is double. How many branch points does the holomorphic covering $f : \mathbb{C} \to \hat{\mathbb{C}}$ possess?

Exercise 2. (7 points)

Let S be a compact polyhedral Riemann surface and let $V(S)$ be the vertex set of S. The *discrete Gaussian curvature* in $P \in V(S)$ is defined as $K(P) = 2\pi - \sum_i \alpha_i$, where the α_i are the interior angles of faces of S adjacent to P.
Prove the *discrete Gauss-Bonnet formula*

$$\chi(S) = \frac{1}{2\pi} \sum_{P \in V(S)} K(P).$$

Exercise 3. (6 points)

Consider a $4g$-gon G_g where opposite edges are topologically identified in such a way that the orientations of these two edges with respect to G_g are different.

(a) Show that any compact Riemann surface of genus g can be represented as such a $4g$-gon G_g.

(b) How can one obtain the $4g$-gon F_g with edges identified as in the lecture from G_g using moves of cutting and glueings?

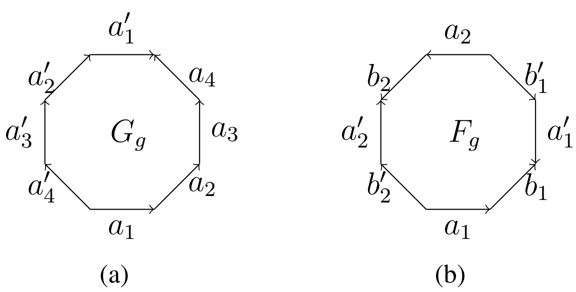

Figure 1: Polygons G_g and F_g for $g = 2$

Due: Tutorial on 14 November, 2012.