TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

Exercise Sheet 7

Exercise 1. (7 points)

Let $\mathcal{R}$ be the compact Riemann surface associated to the hyperelliptic curve

$$\mu^2 = \prod_{k=1}^{N} (\lambda - \lambda_k),$$

where $N = 2g + 1$ or $N = 2g + 2$.
Construct $2g$ linearly independent real-valued harmonic differentials on $\mathcal{R}$.

Exercise 2. (6 points)

Show that any canonical divisor on a complex torus is principal.

Exercise 3. (7 points)

Let $\mathcal{R}$ be a compact Riemann surface and denote by $H^1_{dR}(\mathcal{R}, \mathbb{C})$ the factor space of closed one-forms modulo exact one-forms. Let H denote the complex vector space of harmonic differentials on $\mathcal{R}$.
Show that the canonical map $H \to H^1_{dR}(\mathcal{R}, \mathbb{C})$ is an isomorphism.

Due: Tutorial on 12 December, 2012.