TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Complex Analysis II / Geometry III - WS 12
Riemann Surfaces: Classical and Discrete

www3.math.tu-berlin.de/geometrie/Lehre/WS12/ComplexAnalysisII/

Exercise Sheet 9

Exercise 1. (6 points)

Let $\mathcal{R}$ be a compact Riemann surface of genus $g > 0$ and let $P \in \mathcal{R}$. Denote by N the largest positive integer such that there exist no meromorphic function on $\mathcal{R}$ having one pole of order N at P and no further poles. Show that P is a Weierstraß point if and only if $N > g$.

Exercise 2. (12 points)

Consider an octagon $G_2 \subset \mathbb{C}$ with vertices on the eighth roots of unity where opposite edges are identified by the corresponding translations. We equip the corresponding compact Riemann surface $\mathcal{R}$ of genus 2 with the complex structure inherited from $\mathbb{C}$.

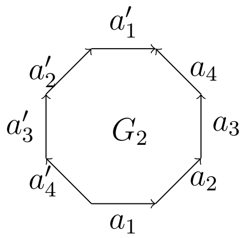

Figure 1: Octagon G_2

a) Let P be the point on $\mathcal{R}$ corresponding to the vertices of G_2. Construct a basis of either $L(-2P)$ or of $H(2P)$ and conclude that P is a Weierstraß point.

b) Find a holomorphic involution of $\mathcal{R}$ having exactly six fixed points, and show that these six points are the Weierstraß points of $\mathcal{R}$.

Exercise 3. (2 points)

a) Enjoy Christmas!

b) Have a Happy New Year!

Due: Tutorial on 16 January, 2013.