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http://www3.math.tu-berlin.de/geometrie/Lehre/WS12/DiffGeoII WS 12

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 6

(Orientation, Riemannian manifolds)

due 28.11.2012

Exercise 1 **5 points**

For which $n \in \mathbb{N}$ is $\mathbb{RP}^n$ orientable, and for which not? Prove it.

Exercise 2 **5 points**

Prove that an n–dimensional manifold M for which there exists an immersion $f\colon M \to \mathbb{R}^{n+1}$ is orientable if and only if there is a smooth nowhere–vanishing normal vector field along (M, f), i.e. a map $N\colon M \to \mathbb{S}^n$ such that $N(p) \perp d_p f(T_p M)$ for all $p \in M$.

Exercise 3 **5 points**

Consider $\mathbb{S}^2 \subset \mathbb{R}^3$ with its induced metric. Show that the following maps are isometric immersions:

a) $f_1\colon \mathbb{S}^2 \to \mathbb{R}^6, \quad (x, y, z) \mapsto \left(\frac{1}{\sqrt{2}}x^2, \frac{1}{\sqrt{2}}y^2, \frac{1}{\sqrt{2}}z^2, xy, xz, yz\right),$

b) $f_2\colon \mathbb{R}^2 \to \mathbb{R}^4, \quad (u, v) \mapsto (\cos u, \sin u, \cos v, \sin v).$

In how far can f_1 be considered as isometric immersion from the real projective plane $\mathbb{RP}^2$ to $\mathbb{RP}^5$, and f_2 as one from the 2–torus $T^2 = \mathbb{R}^2/2\pi\mathbb{Z}^2$.